Measurement of the Total Transpiration from a Forest.

by Yoshito YAMAOKA

Part 1. Meaning of the total transpiration from a forest as a fundamental datum of water resources.

1. Introduction

The estimation of water quantity of a weir is one of the important problem on design in building a water-power-plant. However, the estimation of water quantity up to this time is not yet satisfactory in its accuracy. For example, in the case of constructing a weir for water-power-plant of Kōtō-river, Yamaguchi prefecture, Japan, which had been completed in 1950, there was considerable discrepancy among the experimental and the estimated water quantity calculated by the quantity of rainfall, evaporation and the water soaked into the ground, in the catchment area of 324 square kilometers. Accordingly, the estimated water quantity had to be corrected several times, and the correction amounted at least from 200% to 300% of the estimated value. What was responsible for this discrepancy amounting to such a large quantity? Let us consider next the reason why this large discrepancy arose.

2. Causes of the discrepancy.

Let W represents the quantity of rainfall in a catchment area within a certain period, W_1 the quantity of underground water which flows out from the catchment area, W_2 the sum of water losses due to transpiration and interception of a forest, and W_3 the water loss by evaporation from the ground, then the quantity of effective water flow W_0 can be expressed in the following equation.

$$W_0 = W - W_1 - W_2 - W_3$$

where W_0 and W_3 are the measurable water quantity and W the known water quantity. But it is difficult to measure the water quantity W_1 which varies with the geological conditions of the soil. And also W_2 is a water quantity difficult to measure in its natural state. That is to say, the causes making the estimation of effective water quantity inaccurate lies on the point that two unknown water quantities are included in one equation. By this reason it would be much useful in designing a weir in the future if one of these unknown quantities W_1 and W_2 had been known. So the author has made his plan to measure the quantity W_2.

3. Necessity of leaf area measurement.

As described above, W_2 includes two of the water losses; one of which is the water loss by transpiration from a forest and the other is the interception of rainfall by a forest. According to the reports of Dr. M. Nakayama and Mr. T. Tazaki[1,2], the interception of rainfall by a forest amounts to 1 mm. of rainfall in maximum and in general directly proportional with the leaf area.

The total transpiration from a forest is often estimated by the transpiration per unit leaf weight of green leaf or per unit leaf area. The same method was adopted in the recent reports by Dr. M. Nakayama and Mr. M. Kadota[3,4]. From these reasons it is one of the most important matters to obtain the total leaf area of a forest at first.

There are various convenient methods of measuring the total leaf area of a tree which had been adopted in the past, such as the sampling method (estimating the total leaf area of a tree from leaf area of a typical twig.), method of estimation by the total leaf weight and the leaf area per unit leaf weight, method of estimation by the projection area of crown, etc. Recently, Prof. J. Kittredge[5,6] reported that the following equation holds between the leaf weight W and the stem diameters of trees.

$$\log W = b \log D - a$$

Where a and b represent two constants. By this equation the total leaf weight can be easily obtained, and the estimation of total leaf area of a tree becomes extremely simple.

4. Possibility of estimating the total leaf area of a forest.

The author continued his measurement with respect to the relation of the total leaf area A to the sectional area of stem of a tree since June 1948, and

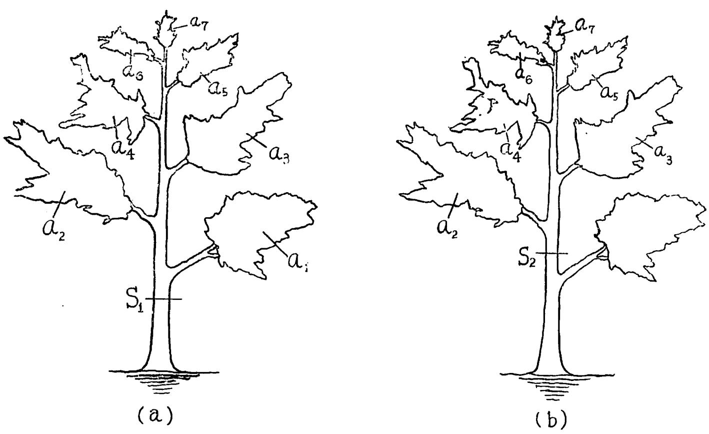

Fig. 1. Sketches of a tree with foliage, showing the measuring method of leaf area and stem sectional area. (a) and (b) show the same tree.

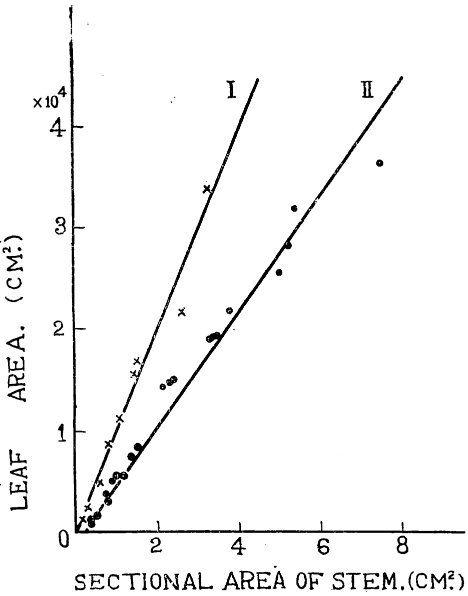

Fig. 2. Relation between leaf area and sectional area of stem of a tree. Line I, Mallotus Japonicus Muel, Arg.; Line II, Podocarpus Macrophyllus D. Don.

obtained the results as shown in fig. 2 (only few examples are shown in the figure) based on 44 samples. As time for measurement, the author chose July and August, or thereabouts. Fig. 2 shows the results obtained as follows.

Let fig. 1 (a) and (b) indicate the schematic diagram of the same tree. And S_1, S_2, ······, represent the minimum sectional areas of stem between successive branches, a_1, a_2, ····, the leaf areas on each branches. Then the following relat on exists among these values.

$$\frac{a_1+a_2+\cdots}{S_1}=\frac{a_2+a_3+\cdots}{S_2}=\frac{a_3+a_4+\cdots}{S_3}=\cdots\cdots=k_0$$

Throughout the investigatin, the leaf area has been measured by a planimeter. The value of k_0 remains nearly constant all along the stem of a tree. And the values of k_0 for numerous trees trends to have a statistically constant value as shown in fig. 3.

Consequently, estimation of the total leaf area of a forest can be made only by measuring the sectional area of stem of trees, if the values k_0 of various species have been known.

The author wishes to express his gratitude to Prof. Dr. U. Nakaya of Hokkaido University who kindly took a great deal of pains to help the author throughout the research.

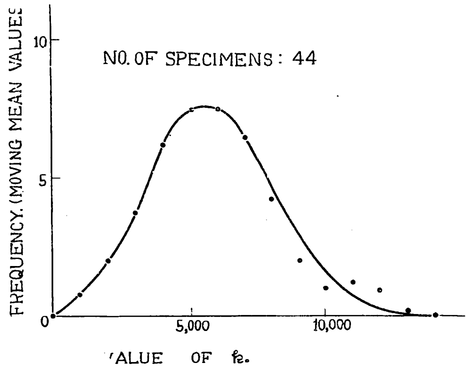

Fig. 3. Frequency curve of k_0.

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(1) Nakayama, M. and Tazaki, T. 1949. Preliminary report on the canopy interception of forests. The Bulletin of the Physiolographical Sci. Res. Inst. No. 2. March: pp. 18—29.

(2) Tazaki, T. 1950. On the canopy-interseption of forests. (II) The Bulletin of the Physiographical Sci. Res. Inst. No. 4. March: pp. 49—56.

(3) Nakayama, M. and Kadota, M. 1948. The influence of the wind on the transpiration of some trees. (No. 1). The Bulletin of the Physiolographical Sci. Res. Inst. No. 1. Jan: pp. 17—34.

(4) Nakayama, M. and Kadota, M. 1949. The influence of the wind on the transpiration of some trees. (No. 2). The Bulletin of the Physiolographical Sci. Res. Inst. No. 2. pp. 10—17.

(5) Kittredge, J. 1944. Estimation of the Amount of Foliage of Trees and Stands. Journ. Forestry, 42, pp. 905—912.

(6) Kittredge, J. 1948. Forest Influences. New York.

要　　約

ある集水区域から流出する有効流量を降雨量から算出するには，その集水区域内の森林の通発によつて失われる全水量を測定することが不可欠であることを述べ，それには先づ森林全体の全葉面積の推定が必要欠くべからざる問題であることを説明した。そして更に個々の立木については，その幹断面積とそれが保持する葉面積とが比例関係にあり，その比例常数は多数の立木について統計的に正規分布をすることから，それによつて森林の全葉面積推定の可能性が考えられることを解いた。

Part 2. Relations between stem and twigs of a tree.

1. Introduction.

The author has described in his former report[1] that the certain rerationship holds between the leaf area and the sectional area of stem of a tree. But the ratio k_0 of the leaf area to the sectional area of stem does not always maintain exactly the same value according to species. So the author investigated furthermore into the physiological relationship of the stem to the twigs of a tree, in order to clarify the cause of variation of the value k_0. As a result, a few facts of the relationship were obtained as follows.

2. Definition of the sizes of twigs and stem.

The circumferential length (or the diameter) of twig generally decreases exponentially after branching from its stem as shown in fig. 1, as far as we investigated.

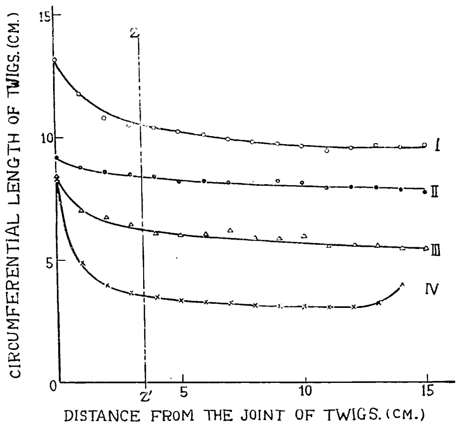

Fig. 1. Relation between the sizes of twigs and the distances from their joints. Curves I & IV: Twigs of Pinus Thunbergii Parl.; Curves II & III: Twigs of Quercus Glauca, Thunb.

Or in some cases, as in Maba Buxifolia Pers., for example, the circumferential length suddenly decreases in a few cm. distance from their joints. In fig. 1 the curve IV trends to increase again at its extreme right end, due to its second branching of the twig. The degree of this decreasing tendency of circumferential length varies with the species, sizes of twigs, etc. So the author has defined the sizes of twigs by the circumferential lengths (or diameters) at the spots where the falling curves tends to become nearly parallel for all cases. In fig. 1, for example, ZZ'-line at about 35 mm. from their joints indicates this position.

The size of stem also varies to some extent between its successive branches, especially in the vicinity of branches. So the author defined its size at the positon where the diameter seems to be minimum between the two successive branches. This position always exists a little below the upper branch.

3. Relation between circumferential lengths of twigs and stem.

Now let fig. 2 (a) and (b) indicate the schematic diagram of the same tree. And let N_1, N_2, ……, indicate the minimum circumferential lengths of stem between successive branches and n_1, n_2, ……, the circumferential lengths of twigs. Then a regular relation, as below, holds for the most part of trees.

$$\frac{n_1+n_2+\cdots}{N_1}=\frac{n_2+n_3+\cdots}{N_2}=\frac{n_3+n_4+\cdots}{N_3}=\cdots\cdots=\frac{1}{a},$$

where the value a is nearly constant throughout the whole stem. Curves in fig. 3 shows the relations for five trees, for example.

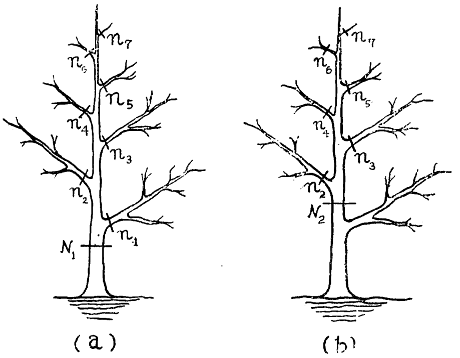

Fig. 2. Sketches of a tree without foliage, showing the measuring method of circumferential lengths of stem and twigs. (a) and (b) show the same tree.

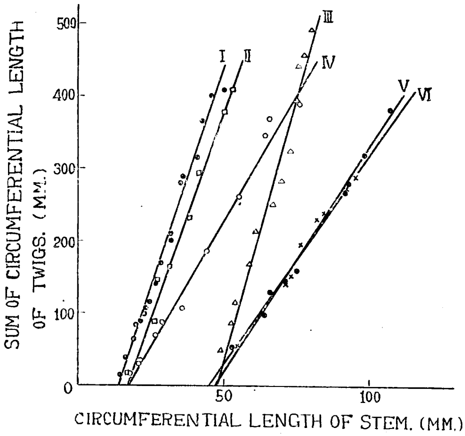

Fig. 3. Relation between circumferential lengths of stem and twigs. Line I & II : Podocarpus Macrophyllus D. Don.; Line III : Juniperus Chinensis L.; Line IV, Machilus Thubergii Sieb. et Zucc.; Line V : Acer ornatum var. Matsumurae; Line VI : a twig of Acer ornatum var. Matsumurae.

The abscissa and the ordinate represent the circumferential lengths of stem Nn and twigs Σn_n, respectively. The points where these curves cuts the abscissa widely differ in this figure, but it is not so important in this place, for it is only due to the fact that the measurements stopped in the way to tips of trees.

The author investigated repeatedly on this relation for 104 broad-leaf trees, and 91 conifers and the same relation appeared almost without exception in these. But, it must be noticed that in the case of dense forest trees or diseased trees the linear relation does not always hold.

4. Distribution of the value 1/a.

Furthermore, our attention should be called upon the gradients of curves in fig. 3, where the gradient 1/a for conifers amounts to about 12, and for the broad-leaf trees 6. The author traced the frequency curves of values 1/a for 104 broad-leaf trees, and 91 conifers respectively, and has obtained the results as shown in fig. 4.

By examining these curves it will be easily seen that the each curve represents the normal distribution curve. And the mean values of 1/a were 12

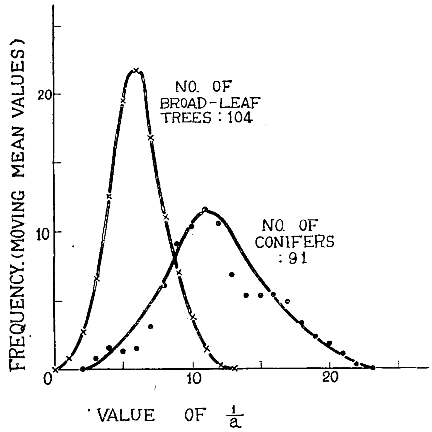

Fig. 4. Frequency curves of 1/a for 104 broad-leaf trees and 91 conifers, respectively.

in conifers and 6 in the broad-leaf trees. It is beyond our comprehension why this relationhip holds against living trees, but we can say at least that the twigs have a certain nature of 1/12 parts of the stem in conifers and in the broad-leaf trees 1/6 parts.

5. Relation between the sectional area of stem and twigs.

In the same manner as described above, the relation between the sectional area of stem and twigs were investigated from the same data once used above. Following are the results.

Let S_1, S_2, ······ be the sectional areas of stem between each successive branches, and s_1, s_2, ······ be the sectional areas of each twigs numbering from the lower side of crown as shown in fig. 5.

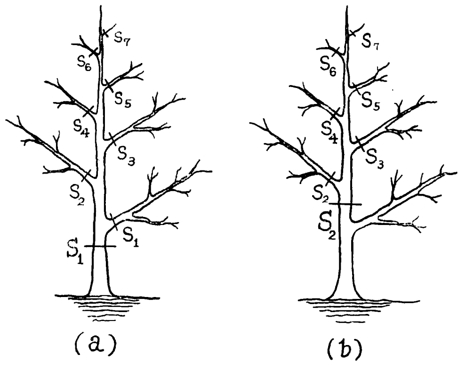

Fig. 5. Sketches of a tree without foliage, showing the measuring method of sectional areas of stem and twigs. (a) and (b) show the same tree.

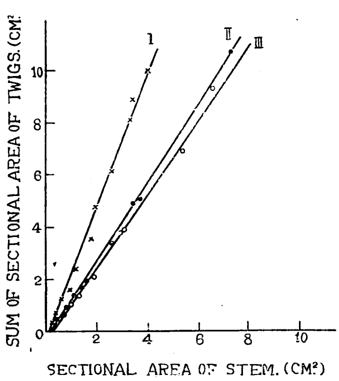

Fig. 6. Relation between the sectional areas of stem and twigs. Line I : Cryptomeria Japonica D. Don.; Line II & III : Machilus Thunbergii Sieb. et Zucc.

Then the following relation, similar to the circumferential relation, holds.

$$\frac{s_1+s_2+\cdots\cdot}{S_1}=\frac{s_2+s_3+\cdots\cdot}{S_2}=\cdots\cdots\cdots=\frac{1}{c}$$

This time the value 1/c does not always maintain constancy well along the stem, but for the present purpose let us consider this value as approximately constant for awhile. Curves in fig. 6 show the relations against three trees, representing two species, for example. While these examples shown in fig. 6

represent satisfactory linear relationship, it is not always the case.

6. Relation between the values of 1/a and 1/c.

The values of 1/c for 104 broad-leaf trees 91 conifers and were calculated as above. And the relation of these values to the corresponding values of 1/a was investigated. Fig. 7 and fig. 8 show these relations.

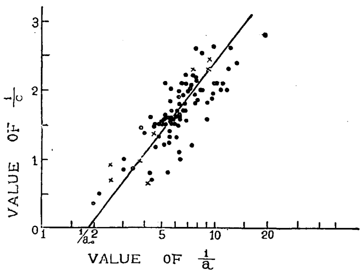

Fig. 7. Relation between two factors 1/a and 1/c, for the broad-leaf trees. x marks are for pine-trees.

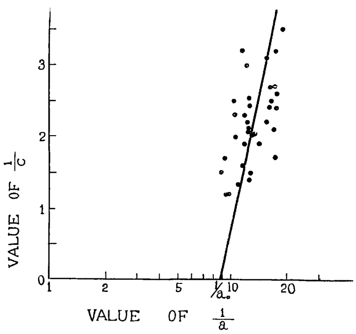

Fig. 8. Relation between two factors 1/a and 1/c, for conifers. (excluding pine-trees)

It can be seen in these figures that the logarithmic values of 1/a holds linear relation with the linear values of 1/c. And this linear relation holds individually for conifers and the broad-leaf trees, respectively. Only pine-trees are the exception as far as the author has investigated. It is shown in the curve of broad-leaf trees (fig. 7) with x marks. Expressing the values of 1/a with $1/a_0$ for the points where these curves cut the abscissa, we have the following equation.

$$\frac{1}{c}=\frac{1}{g}\log\frac{a}{a_0},$$

where g is a constant which differs in conifers from the broad-leaf trees, as shown in fig. 7 and fig. 8.

In conclusion, we can say that the circumferential length (or diameter) and the sectional area of twigs and stem of a tree maintains a certain equilibrium as described above, and accordingly, they can not take any size at will.

This nature of a tree, which combines the circumferential length (or diameter) with the sectional area of twigs and stem, seems to relate to the value of k_0, which is the ratio of the total leaf area to the stem sectional area of a tree.

The author wishes to express his gratitude to Prof. Dr. U. Nakaya of Hokkaido University who kindly took a great deal of pains to help the author throughout the research.

Literature cited.

(1) Yamaoka, Y. 1952. Measurement of the total transpiration from a forest. 1. Meaning of the total transpitation from a forest as a fundamental datum of water resources.

要　　約

前論文[1]で立木の全葉面積と幹断面積とが比例関係にあることを述べたが，その比例常数が同一樹種に対しても正確に一致しないので，幹断面積以外の他の因子が関係しているのではないかと考えて幹と枝との相互関係について調べてみた。その結果１本の立木の幹周辺長と枝周辺長の和との間，及び幹断面積と枝断面積の和との間には夫々単独に一定の関係が保たれていることが判つたので，それについて報告した。また同時にこの両関係を結びつけている他の１つの関係が存在することをも述べた。

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(1) Yamaoka, Y. (1952) Measurement of the total transpiration from a forest. Part 1. Meaning of the total transpiration from a forest as a fundamental datum of water resources.

Part 3. A method of estimation of the total leaf area of a tree.

1. Introduction.

The values of k_0 which represents the ratio of the total leaf area to the sectional area of stem does not only coincide for different trees of the same species but also for different kinds of species, as described in the former report[1]. And the author also considered that the relations of stem to twigs of a tree might be correlated to the values of k_0. So the author investigated into this correlation. And the results were reported in the former report[2]. Now let us investigate further the relationship between these factors and the values of k_0.

2. Relation between k_0 and 1/a, 1/c.

According to the former report[1], the leaf area " a " of a tree correlates to the stem sectional area s in the following form.

$$a = k_0 s \cdots\cdots (1),$$

where k_0 is a constant.

And also according to the former report[2], 1/a, (the ratio of the circumferential length, or diameter, of stem to the circcmferential length, or diameter, of twigs) correlates to 1/c (the ratio of the stem sectional area to the sectional area of twigs) in the following form.

$$1/c = 1/g \ (\log a/a_0) \cdots\cdots (2),$$

where g is a constant which differs in conifers from the broad-leaf trees. From this equation we can calculate the value of 1/c theoreticaily by substituting the experimental value of 1/a in the equation (2), and vice versa. There exists somehow discrepancy among the experimental values and the calculated values of 1/c. Therefore, let us now represent the experimental values of 1/c in $1/c_e$, and the calculated values in $1/c_c$. And let us put $\triangle \frac{1}{c}$ as following.

$$\triangle \frac{1}{c} = \frac{1}{c_e} - \frac{1}{c_c} = \frac{1}{c_e} - \frac{1}{g} \log \frac{a}{a_0} \cdots\cdots (3)$$

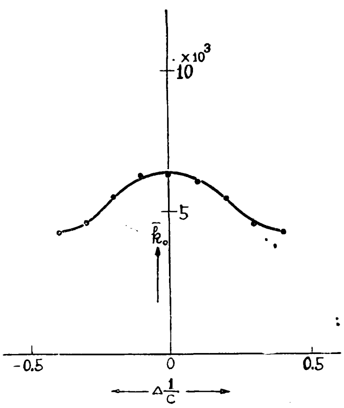

Fig. 1. Relation between $\triangle \frac{1}{c}$ and the mean values of k_0 included in each steps of $\triangle \frac{1}{c}$. Steps of $\triangle \frac{1}{c} = 0.1$.

Then, we can see a certain relation holds betw.en $\triangle \frac{1}{c}$ and k_0, as shown in fig. 1, from the results of 32 broad-leaf trees.

The abscissa represents the value of $\triangle\frac{1}{c}$ in steps of 0.1 and the ordinate the moving mean values of $\bar{k}_0$ which are the mean value of k_0 included in each steps. From this relation, we can express the value of $\bar{k}_0$ in the following functional relation with respect to a and c_e.

$$\bar{k}_0 = f(\triangle\frac{1}{c}) = F(a, c_e)$$

Accordingly, the total leaf area A of a tree can be expressed in the following relation.

$$A = \bar{k}_0 \cdot S = F(a, c_e) \cdot S$$

If the functional form of this equation has been determined against each species, we can easily determine the total leaf area A of a tree more accurately than described in the former report[1] by measuring the values a, c_e and S. And as we have already shown an example in fig. 1 with respect to 32 broad-leaf trees, the functional form of the broad-leaf trees will be easily determined, if desired.

In conclusion, the author wishes to express his gratitude to Prof. Dr. U. Nakaya of Hokkaido University who kindly took a great deal of pains to help the author throughout the research.

Literature cited.

(1) Yamaoka, Y. 1952. Measurement of the total transpiration from a forest. (1) Meaning of the total transpiration from a forest as a fundamental datum of water resources.

(2) Yamaoka, Y. 1952. Measurement of the total transpiration from a forest. (2) Relations between stem and twigs of a tree.

要　　約

立木の幹及び枝の周辺長並びに断面積は夫々一定の比例関係を保ち，更に両関係を結びつける１つの関係が存在することを前論文[1]で述べた。これが立木の葉面積と幹断面積との比の値に如何に影響するかについて本報告では論述した。実測された幹と枝との関係が幹及び枝の周辺長の関係及び断面積の関係を互いに結びつけている釣合条件から外れる度合が大きければ大きい程，その立木の幹の単位断面積毎に保有する葉面積が減少することを明らかにした。従つてこの関係を用いて，より正確に立木の全葉面積を推定することが可能になる。

(1) Yamaoka, Y. (1952). Measurement of the total transpiration from a forest. Part 2. Relations between stem and twigs of a tree.

Part 4. A method of estimation of the total leaf area of a forest.

1. Introduction.

It is indispensable to obtain the total leaf area of a forest before we start to measure the total transpiration from a forest. By this reason the auther will describe below a method of estimation of the total leaf area of a forest. And as it is convenient in the explanation, let us denote the various amount of total leaf area as follows:

a = area of individual leaf

A = total area of one tree = Σa

ΣA = total area of a forest = $\Sigma\Sigma$a

In the former reports[1][2][3], theauther has discussed the correlations between the total leaf area A and the various factors of individual tree and also described a method for estimating the total leaf area A of individual tree. But now we will discuss the various factors which correlates to the leaf area of a forest of the same species, and with regard to it we will describe a method of estimation of the total leaf area of a forest.

All of the experimental data mentioned in this report have been surveyed in Yamaguchi district, Japan.

2. Measuring methods of the total leaf area.

One of the most laborious work in this research was the measurement of the total leaf area A. It is not so difficult to measure the total leaf area of young trees several years old, but in ages of 20 to 30 or more it is almost impossible to measure the total leaf area in practice. In such a case we are obliged to use some convenient methods. Two of the convenient methods adopted in this research are as follows:

1. After counting the whole number of leaves an appropriate number of leaves were picked up by the sampling method, and their leaf areas were measured. According to the linear relationship of the leaf area to its number, we estimated the total leaf area A.

2. According to the linear relationship of the leaf area to its weight, we estimated the total leaf area A by measuring the total leaf weight and the leaf area per unit weight.

The former method was applied to measure the broad-leaf trees and the latter to the conifers. It is because the leaf number of broad-leaf trees is by far smaller than the conifers and it is not so difficult to count its whole number of leaves, but it is opposite to the above in the case of conifers.

Furthermore, as we have found out the fact that the leaf number carried in group on "the smallest twings" are statistically constant in the case of the first method, it became possible to estimate the whole number of leaves only by counting the total number of these smallest twigs. "The smallest twigs" means the small twigs which have branched terminally. (see fig. 1) Applying

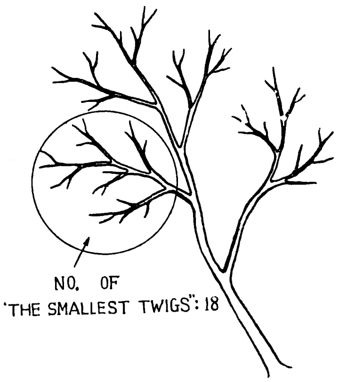

Fig. 1. Sketch of a twig showing the number of "the smallest twigs". 18 smallest twigs are included in the circle.

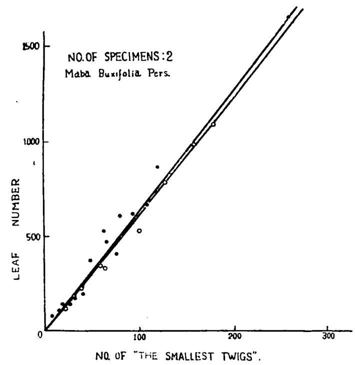

Fig. 2. Relation between the number of leaves and the smallest twigs.

this relation we can save our trouble by several times than before. As an example of the smallest twigs carrying statistically constant number of leaves, we have mentioned the case of Maba Buxifolia Pers. in fig. 2. The leaf number of the smallest twigs are in close linear relation as it is shown in fig. 2. Fig. 2 includes the results obtained from two of Maba Boxifolia Pers.

Measurement of the leaf area "a" of the broad-leaf trees was conducted with a planimeter. And as the leaf area "a", only one side of the two leaf surfaces was counted. In order to measure the leaf area "a" of conifers, especially of pine trees, we placed a pair of long leaf together and treated its sectional form as an ellipse. Its major- and minor-diameter were measured with a dial indicator. The surface area of this pair of leaves was determined by adding twice of the minor-diameter to its circumferetial length and multiplying it by the total length of leaf.

3. Relation between total leaf area and age.

Relation of the total area, which have been obtained as above, to ages of trees was investigated. Fig. 3 shows the results obtained from 19 pine-trees. Those ages are spreading over from 2 to 21 years. We have also measured about 11 Machilus Thunbergii Sieb. et Zucc. of ages from 4 to 19 years, and their results are indicated in fig. 4. Also similar relation holds against other species.

But it is disadvantageous to estimate the total leaf area ΣA of a forest from the relation of the total leaf area to the age, for the dotted marks are so

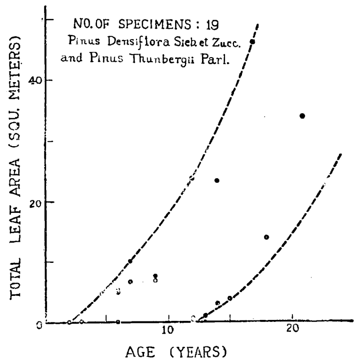

Fig. 3. Relation between the total leaf area and age, for Pinus Densiflora Sieb. et Zucc. and Pinus Thunbergii Parl. Three dotted points on the abscissa show the total leaf areas less than 0.1 squ. meters.

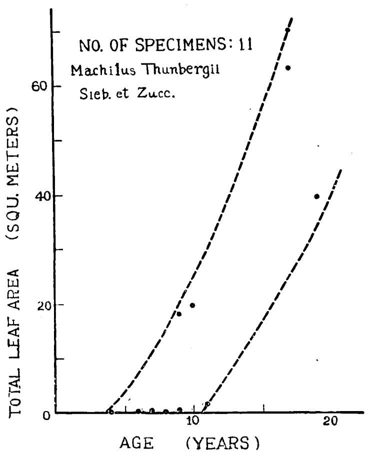

Fig. 4. Relation between the total leaf area and age, for Machilus Thunbergii Sieb. et Zucc. Three dotted points on the abscissa show the total leaf areas less than 0.5 squ. meters.

widely spread over the diagram as it can be seen at a glance.

4. Relation between total leaf area and sectional area of stem.

Since it was known that the estimation of the total leaf area based on age is impossible as described above, the auther searched for other appropriate factors useful for this purpose. As we considered the sectional area of stem just under the crown might be utilized for this purpose, the relation of the total leaf area A to the sectional area of stem just under the crown was investigated. Investigating against the pine-trees and Machilus Thunbergii Sieb. et Zucc., those having already been described above, we obtained fig. 5 and fig. 6. In

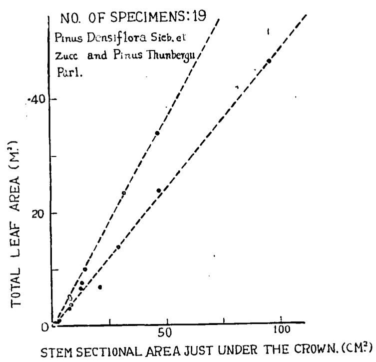

Fig. 5. Relation between the total leaf area and the stem sectional area just under the crown. Pinus Densiflora Sieb. et Zucc. and Pinus Thunbergii Parl.

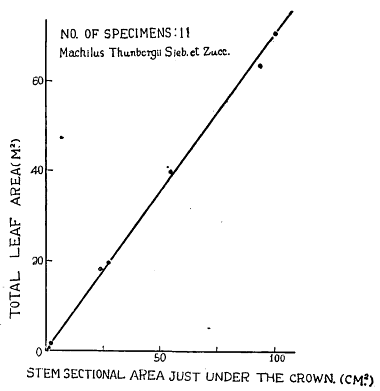

Fig. 6. Relation between the total leaf area and the stem sectional area just under the crown. Machilus Thunbergii Sieb. et Zucc.

this case we can find the remarkably close correlation between them as compared with the former case. That is to say, the linear relationship holds generally between the total leaf area A and the sectional area of stem just under the crown of trees. This result will give rise of a new method in the field of practical method of measuring the total leaf area. Now let d_0 represents the stem diameter just under the crown, and k one of the constants proper to the species, then the total leaf area A can be expressed in the following form.

$$A = k . \frac{\pi}{4} d^2_0 \quad \cdots\cdots (1)$$

5. Relation between stem diameter just under the crown and breast-height diameter of stem.

As we have already described, a linear relationship holds between the total leaf area A and the sectional area of stem just under the crown. But in order to estimate the volume of timber of a forest we generally measure, in Japan, the so-called breast-height diameter which corresponds the height of 1.2 meters from the ground. Accordingly, it is convenient to know the relation of the total leaf area A to the breast-height diameter in order to estimate the total leaf area of a forest from the breast-height diameters, number of trees, volume of timber. etc. Or, in other way, if the relation of the breast-height diameter to the stem diameter just under the crown is known, it will serve for the same purpose. So we investgated the relation of the breast-height diameter to the stem diameter just under the crown with respect to various species and obtained the results shown in fig 7. Six species are included in this diagram, Japanese black- and red-pine trees, Machilus Thunbergii Sieb. et Zucc., Maba Buxifolia Pers., Eurya Japonica Thunb., oak trees, Pasania Cuspidata. A certain linear relation holds almost with no connection to species. From this result we may conclude without uncertainty that the linear relation holds fairly well between the stem diameter just under the crown and the breast-height diameter. Now if d represents the breast-height diameter and d_0 the diameter just under the crown of trees, we have the following equation.

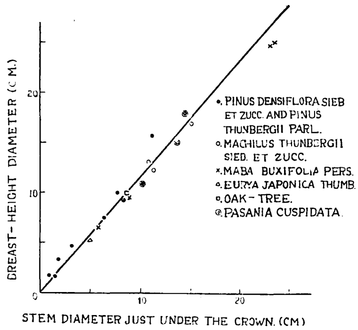

Fig. 7. Relation between the breast-height diameters and the stem diameters just under the crowns.

$$d = m d_0 \quad \cdots\cdots (2)$$

Where m is a constant. And this can be considered as an unrelated constant to species. By substituting the equation (2) into the equation (1), the relation of the total leaf area A to the breast-height diameter d can be represented as the following equation.

$$A = \frac{k}{m^2} \cdot \frac{\pi}{4} d^2 \quad \cdots\cdots (3)$$

Consequently, the total leaf area A of a forest can be obtained by the following equation.

$$\Sigma A = \frac{k}{m^2} \cdot \frac{\pi}{4} \cdot \Sigma d^2 \quad \cdots\cdots (4)$$

6. Distribution of breast-heit diameters of trees of the same age.

So far as we mentioned above, the total leaf area is closely correlated to the breast-height diameter. So we should investigate next the distribution of the breast-height diameters of trees of the same age. The auther has measured 913 Cryptomeria Japonica D. Don. of 40 years in Namera national forest, compartment 27, in Yamaguchi district, Japan. The frequency curve with respect to this results is shown in fig. 8 classifying the diameter in 2 cm steps.

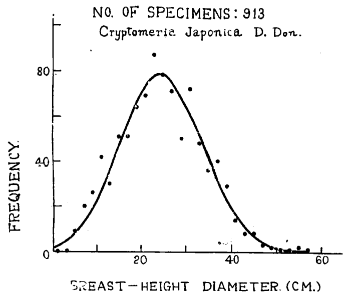

Fig. 8. Frequency curve of the breast-height diameters of 913 Cryptomeria Japonica D. Don., 40 years of age, in Namera National Forest, Yamaguchi Prefecture, Japan.

Dotted marks in the figure show the experimental values and the curve has been traced by the following empirical formula

$$y = 78.20e^{-0.00576} \quad (x - x_m)^2,$$

where x_m represents the value of x at maximum frequency.

The distribution curve of the breast-height diameters of forest trees of the same age is in accordance with the Gauss' distribution law as shown.

7. Estimation of total leaf area ΣA by number of trees and mean breast-height diameter.

Since the distribution of breast-height diameters obeys the Gauss' distribution law as described above, the total leaf area of a forest can be obtained as follows when the total number N of trees and the mean breast-height diameter d_m of a forest are known.

Now let the abscissa represents the breast-height diameter d of trees and the ordinate the frequency and trace a frequency curve as shown in fig. 9. Then the following equations represent the relation of n to d, since the curve obeys the Gauss' distribution law.

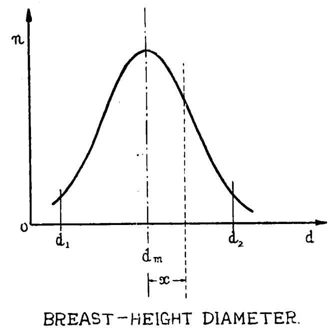

Fig. 9. Frequency curve of the breast-height diameters of trees. (Gauss' distribution curve)

$$\left.\begin{array}{ll} n=\dfrac{N\triangle d}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{x^2}{2\sigma^2}} & \triangle d:\ \text{sections of diameter} \\ \sigma^2=\dfrac{1}{N}\sum_i\{(d_i-d_m)^2 n_i\}=\dfrac{1}{N}\sum_i(x_i^2 n_i)\cdots\cdots\cdots & \text{standard deviation} \\ x=d-d_m & \end{array}\right\}\cdots\cdots(5)$$

Where $\triangle d$ represents the breadth of a section of diameter d and it can be considered as a constant. In order to obtain the sum of sectional area of stems existing in a forest by the equation (5), we should calaulate at first the value of $\frac{\pi}{4}\sum_i n_i d_i^2$, that is to say, the value of $\sum_i n_i d_i^2$. (see fig. 9)

$$\sum_i n_i d_i^2=\sum_{d_1-d_m}^{d_2-d_m}\frac{N\triangle d}{\sqrt{2\pi}\,\sigma}\cdot e^{-\frac{x^2}{2\sigma^2}}(d_m+x)^2$$

$$=d_m^2\cdot\frac{N\triangle d}{\sqrt{2\pi}\sigma}\sum_{d_1-d_m}^{d_2-d_m}e^{-\frac{x^2}{2\sigma^2}}+2d_m\cdot\frac{N\triangle d}{\sqrt{2\pi}\sigma}\sum_{d_1-d_m}^{d_2-d_m}x\cdot e^{-\frac{x^2}{2\sigma^2}}+$$

$$\frac{N\triangle d}{\sqrt{2\pi}\sigma}\sum_{d_1-d_m}^{d_2-d_m}x^2\cdot e^{-\frac{x^2}{2\sigma^2}}\cdots\cdots\cdots\cdots(6),$$

where,

$$N=\sum_i n_i=\frac{N\triangle d}{\sqrt{2\pi}\sigma}\sum_{d_1-d_m}^{d_2-d_m}e^{-\frac{x^2}{2\sigma^2}}\cdots\cdots\cdots\cdots:(7)$$

And since,

$$\int_{d_1-d_m}^{d_2-d_m}x\cdot e^{-\frac{x^2}{2\sigma^2}}dx=-\sigma^2\left(e^{-\frac{1}{2\sigma^2}(d_2-d_m)^2}-e^{-\frac{1}{2\sigma^2}(d_1-d_m)^2}\right)$$

We have the following equation approximately:

$$\sum_{d_1-d_m}^{d_2-d_m}x\cdot e^{-\frac{x^2}{2\sigma^2}}\fallingdotseq-\sigma^2\left(e^{-\frac{1}{2\sigma^2}(d_2-d_m)^2}-e^{-\frac{1}{2\sigma^2}(d_1-d_m)^2}\right)\cdots\cdots\cdots(8)$$

Similarly,

$$\int_{d_1-d_m}^{d_2-d_m}x^2 e^{-\frac{x^2}{2\sigma^2}}dx=\sigma^2\left\{(d_1-d_m)e^{-\frac{1}{2\sigma^2}(d_1-d_m)^2}-(d_2-d_m)e^{-\frac{1}{2\sigma^2}(d_2-d_m)^2}\right\}$$

$$+\sigma^2\int_{d_1-d_m}^{d_2-d_m}e^{-\frac{x^2}{2\sigma^2}}dx$$

and the next equation holds approximately:

$$\sum_{d_1-d_m}^{d_2-d_m}x^2 e^{-\frac{x^2}{2\sigma^2}}\fallingdotseq\sigma^2\left\{(d_1-d_m)e^{-\frac{1}{2\sigma^2}(d_1-d_m)^2}-(d_2-d_m)e^{-\frac{1}{2\sigma^2}(d_2-d_m)^2}\right\}$$

$$+\sigma^2\sum_{d_1-d_m}^{d_2-d_m}e^{-\frac{x^2}{2\sigma^2}}\cdots\cdots\cdots\cdots(9)$$

From the equations (7), (8), (9) and (6), we have,

$$\sum_i n_i d_i^2\fallingdotseq N(d_m^2+\sigma^2)+\frac{N\cdot\triangle d\cdot\sigma}{\sqrt{2\pi}}\left\{(d_1+d_m)e^{-\frac{1}{2\sigma^2}(d_1-d_m)^2}\right.$$

$$\left.-(d_2+d_m)e^{-\frac{1}{2\sigma^2}(d_2-d_m)^2}\right\}\cdots\cdots\cdots\cdots(10)$$

Consequently, the equation of the sectional area of stems can be written as below.

$$\frac{\pi}{4}\sum_{i} n_i d_i^2 \doteqdot \frac{\pi}{4}\cdot N\cdot(d_m^2+\sigma^2)+\frac{\sqrt{\pi}\cdot N\cdot\triangle d\cdot\sigma}{4\sqrt{2}}\{(d_1+d_m)e^{-\frac{1}{2\sigma^2}(d_1-d_m)^2}$$

$$-(d_2+d_m)e^{-\frac{1}{2\sigma^2}(d_2-d_m)^2}\} \cdots\cdots\cdots(11)$$

The equation (11) is so complicated that it is inconvenient to our practical use. By examining the equation (11) we can see the value of the second term is so small compared with the first term. A practical example will be shown in the following:

Experimental values on Cryptomeria Japonica D. Don. in Namera National Forest, compartment 27, Yamaguchi Prefecture, Japan, which have once been mentioned above were:

$d_m=24.45$ cm, $N=913$, $\triangle d=2.00$ cm,
$d_1=1.00$ cm, $d_2=57.00$ cm, $\sigma^2=86.80$.

After substituting these values into the equation (11), we have

$$\frac{\pi}{4}\sum_{i} n_i d_i^2=(490906+2363)\ \mathrm{cm}^2 \cdots\cdots\cdots(12)$$

Since the value of the second term amounts to only about 0.48% of that of the first term and it is also within our experimental error, there are no trouble at all to neglect the second term in practice. According to this, the following equation can serve as an equation of calculation of the sum of the total sectional area of stems.

$$\frac{\pi}{4}\sum_{j} n_i d_i^2=\frac{\pi}{4}N(d_m^2+\sigma^2) \cdots\cdots\cdots(13)$$

Again the auther has calculated the breast-height sectional area one by one from the experimental data of breast-height diameter of 913 Cryptomeria Japonica D. Don. of 40 years, surveyed in Namera National Forest, Yamaguchi Prefecture, Japan, and has summed up in order to see to what extent the error between the experimental value and the calculated value of the equation (13) amounts. The value amounted to 490,667 cm^2. This value differs only by 239 cm^2 from the calculated value of the equation (13), namely 490,906 cm^2. Expressing this in percentage error it amounts only to 0.05% excess. By this example it is obvious that the equation (13) can serve in practical use with sufficient accuracy. Finally, from the equations (13), the total leaf area ΣA of a forest of the same age and species can be obtained by the following equation (14).

$$\Sigma A=\frac{k}{m^2}\cdot\frac{\pi}{4}\cdot N\cdot(d_m^2+\sigma^2) \cdots\cdots\cdots(14),$$

where k and m represent two constants, the former concerning to the leaf area and the latter to diameter, N the number of trees in a forest, d_m the mean breast-height diameter and σ the standard deviation of the breast-height diameters.

8. Conclusion.

The total leaf area A of a tree maintains some correlation to its age. However, it is by far small when compared with the correlation of the total leaf area A to the stem sectional area just under the crown. The close correlation of the total leaf area A to the stem sectional area just under the crown is somewhat remarkable.

Furthermore, the stem diameter just under the crown was closely propotional to the breast-height diameter and its proportional constant remains constant among various species as far as we investigated. From this relationship we can represent the total leaf area of trees by means of the breast-height sectional area of their stems.

And also the distribution of breast-height diameters of trees of the same age obeys the Gauss' distribution law.

From these relations the auther has mentioned a method of estimation of the total leaf area of a forest of the same age and the same species by the mean breast-height diameter and total number of trees, and derived the calculation formula of ΣA. The calculated value of this formula differs only by 0.05% excess compared with the sum of breast-height sectional area obtained by the results which had been surveyed against 913 Cryptomeria Japonica D. Don. of 40-years of age. From this results we can say the calculation formula which has been described above can serve in practical use with sufficient accuracy.

In conclusion, the auther wishes to express his gratitude to prof. Dr. U. Nakaya of Hokkaido University who had kindly taken a great deal of pains to help the auther throughout the research and to Prof. Dr. A. Higashi of the same University who had kindly led the author in preparing the report. And also to Mr. S. Tsuboi and Mr. S. Hirasa of the Yamaguchi District forestry office who gave to the auther great convenience in the case of surveying Namera National Forest.

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要　　　約

立木の全葉面積 A と樹齢並びに樹冠直下の幹断面積との間の関係を述べ，立木の全葉面積Aを推定するには樹冠直下の幹断面積を用うるのがより有利であることを説明した。そして樹冠直下の幹直径は胸高直径に密接に比例するから，胸高直径によつて立木の全葉面積 A を表わすことが出来ることを述べた。更に滑国有林における樹齢 40 年の杉 913 本についての胸高直径の実測値から，その度数分布がガウスの分布曲線に従うことを述べ，森林全体の胸高断面積の総和を簡単に求める式を誘導し，実測値と 0.05% 程度の差で一致することを示した。これらの結果から，同樹種の森林の全葉面積 ΣA は，その森林中の立木の平均直径を d_m，直径分布の標準偏差を σ，立木本数を N，樹種に固有な葉面積係数を k，樹種に無関係な常数を m とすると，

$$\Sigma A=\frac{k}{m^2}\cdot\frac{\pi}{4}\cdot N\cdot(d_m^2+\sigma^2)$$

によつて算定し得ることを報告した。