TURBULENCE IN THE LOWEST ATMOSPHERE

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CONTENTS

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Methods of approach and the scope of the present work

One of the characteristics of the atmospheric turbulence is the participation of stability of the air layer in the phenomena, and at the same time it makes the problem more difficult. The effect of stability may be considered by the three methods as follows:

(1) theoretically

Rossby and Montgomery[32], and recently Lettau[27] and Kawahara[20] adopted this method. If the theory is sound, this will give the most reliable results. But at the present stage the theory seems to contain some ambiguities or defects. We shall discuss later some of the equations.

(2) empirically

(a) from the profile of mean wind velocity

Wind velocity is easy to measure and many observations have been made. If the profile is known we can obtain the stability dependence of eddy viscosity, mixing length, etc., using widely accepted relations between them.

(b) from the irregular or turbulent component of wind velocity

Eddy viscosity and mixing length, etc., will be obtained also from the turbulent component of wind velocity. Ertel's[6] formula is well-known, but it has not been utilized to obtain the stability dependence of the eddy viscosity. Lettau[23],[24],[25] used "kinetische Austauschformel" to obtain austausch-coefficient from his free balloon measurements and got some conclusions about the degree of turbulence near the cloud. Recently Frankenberger[9] succeeded to obtain a relation between the eddy viscosity and stability though his results showed some scattering. This method seems to be interesting but rather difficult, and only a few measurements have been made.

The scope of the present work

As the theories are not sound at present we will adopt the empirical methods. In the first part of this paper concerning the profile of mean wind velocity, we will review studies on wind velocity profile hitherto done by many investigators (the author inclusive) and give the most reliable one now obtained, because the primary importance in this method is to get the most exact profile possible, and then consider the stability dependence of eddy viscosity, mixing length, etc.

In the second part of this paper regarding the turbulent component of wind velocity the author will describe his own observations, as only a few experiments in this field have been made. Though his observations do not necessarily give satisfactory results concerning the stability dependence, yet they seem to be interesting in some respects.

PART I Mean wind velocity

§ 1. Wind velocity profile.

The development of research on wind velocity can be conveniently considered dividing into two stages; namely, the former, prewar and war period, in which the question of whether the profile should be represented by the logarithmic or the exponential law constitued the main problem; and the latter, postwar period, in which the existence of deviation from the logarithmic law was recognized with certainty.

(I) Logarithmic law and exponential law

(a) Hellmann's[13),13),14),15),16)] measurement

One of the most detailed study of wind in the lower atmosphere was due to Hellmann, who expressed results up to the height of 258 m. either by logarithmic formula

$$u = a\ln(z+c)+b \qquad (1)$$

or by exponential

$$u = az^{\frac{1}{n}}, \qquad (2)$$

where u denotes the wind velocity at the height z, and a, b, c and n are constants. He observed that both of them represent the results fairly well, but, above all, the exponential profile with

$n=5$ for $z>16$ m.

$n=4$ for $z<2$ m.

was best.

(b) Heywood's[18)] measurement

The dependence of wind velocity on stability was made clear for the first time by Heywood. From measurements of wind velocity at two heights $z=95$ m. and $z=13$ m. with simultaneous temperature measurements, he observed that at constant u_{95}, $u_{95}-u_{13}$ increased from negative to positive values of $T_{87}-T_{13}$, and at constant $T_{87}-T_{13}$, $u_{95}-u_{13}$ increased with u_{95} where T denotes temperature. Fig. 1 and 2 show his results. In fig. 2 he draws curves after Taylor's[59)] theoretical conclusion. But Taylor's theory was based upon assumptions of finite surface wind and constant eddy viscosity, both of which are recognized to be not the case in the lowest layers of the atmosphere.

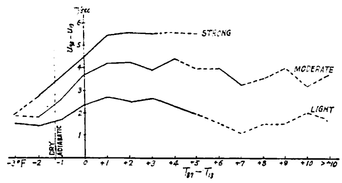

Fig. 1 Difference in wind velocity between 95 m. and 13 m. in relation to difference in temperature between 87 m. and 13 m.

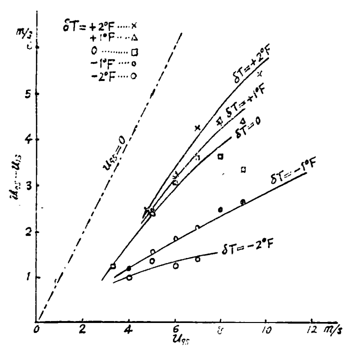

Fig. 2. Difference in wind velocity between 95 m. and 13 m. in relation to wind velocity at 95 m. for fixed values of temperature gradient.

(c) Prandtl's[31)] research

Prandtl applied results of aerodynamics to the lowest atmosphere. The definition of shearing stress τ is

$$\tau = A\frac{\partial u}{\partial z}, \qquad (3)$$

where A is austausch-coefficient. He improved this expression and deduced

$$\tau = \rho l^2\left(\frac{\partial u}{\partial z}\right)^2, \qquad (4)$$

in which only the geometric quantity l (mixing length) is contained except air density ρ. The results of aerodynamics showed that in the absence of stability l was proportional to z and the proportionality factor was equal to 0.4, so (4) becomes

$$\frac{du}{dz} = \frac{2.5}{z}\sqrt{\frac{\tau}{\rho}}. \qquad (5)$$

If τ is assumed to be constant with z in the lowest atmosphere, e. g. lowest 50 or 100 m., (5) is easily integrated and gives

$$u = 2.5\sqrt{\frac{\tau}{\rho}}\, ln\frac{z}{z_0}, \qquad (6)$$

where z_0 is an integration constant which adjust itself that at the surface of the roughness u equals to the actual velocity. z_0 is considered to be in a definite relation to the height of the roughness h and in the aerodynamics it was found that

$$z_0 = \frac{k}{30}, \qquad (7)$$

where k means the diameter of a grain of sand adhered to the wall. Instead of 30, smaller values are found in the atmosphere after that, e. g. 7.35 (Paeschke[28]) and 3 (Shiotani and Yamamoto[36], and Takeda[45]).

Prandtl further made some discussions about the non-adiabatic atmosphere, but without a remarkable conclusion.

The significance of the Prandtl's analysis seems to lie in the fact that in the adiabatic atmosphere the wind profile is shown to be represented by the logarithmic law, and since then the exponential law has been used only in cases where theoretical treatments become especially simple.

(d) Best's[1] measurement

One of the most detailed measurements of wind and temperature in the lowest atmosphere was done by Best. The stability depependence $U(z)$ (wind velocity at height z expressed as percentages of the simultaneous velocity at 1 m.) was given by him as follows:

Table 1.

		2.5cm.	5 cm.	10 cm.	25 cm.	50 cm.	100cm.	200cm.	506cm.
Temperature Gradient	−3° F./m.	42.9	52.0	66.7	81.0	90.3	100	107.1	—
	Zero	36.4	49.0	63.0	79.2	90.3	100	111.7	122.5
	+1° F./m.	33.9	47.8	60.0	77.1	89.5	100	114.3	—

He observed that in the adiabatic atmosphere $U(z)$ is in the linear relation with $\log(z-1)$. In the non-adiabatic atmosphere he could not work with logarithmic law, but adopting exponential law (2) he obtained stability dependence of n as is shown in table 2 for the height interval from 25 cm. to 2 m.

Table 2. Variation of index in exponential law with temperature gradient

Temperature Gradient	Index	Simultaneous Conditions
−3.0°F./m.	7.15	Layer 25 cm. to 2 m.
Zero	5.87	Velocity at 1 m. between 1.5 m./sec.
+1.0°F./m.	5.27	and 4.0 m./sec.

From the table it is seen that the index n decreases as the air layer becomes stable.

(e) Rossby and Montgomery's[32] theory

The first theoretical treatment of the stability dependence of turbulence in the lowest atmosphere was attempted by Rossby and Montgomery. They started from two assumptions:

(I) an energy equation

$$l^2\left(\frac{\partial u}{\partial z}\right)_s^2 = l_s^2\left(\frac{\partial u}{\partial z}\right)_s^2 + \frac{\sigma g}{\theta}\frac{\partial \theta}{\partial z}\, l_s^2, \quad \dots\dots\dots\dots\dots\dots (8)$$

where θ represents the potential temperature and g the acceleration of gravity and suffix s is applied to denote quantities in the stratified atmosphere, and σ is a proportionality factor (40 according to Rossby and Montgomery) which has been called Rossby's constant subsequently,

(II) constancy of shearing stress with stability

$$\sqrt{\frac{\tau}{\rho}} = l_s\left(\frac{\partial u}{\partial z}\right)_s = l\frac{\partial u}{\partial z}, \quad \dots\dots\dots\dots\dots\dots (9)$$

and deduced

$$\frac{\partial u}{\partial z} = \frac{v_*}{k(z+z_0)}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\gamma^2(z+z_0)^2}{\left(\frac{v_*}{k}\right)^2}}}, \quad \dots\dots\dots\dots (10)$$

where

$$v_* = \text{frictional velocity} = \sqrt{\frac{\tau}{\rho}}, \quad \dots\dots\dots\dots\dots\dots (11)$$

and

$$\gamma^2 = \frac{\sigma g}{\theta}\frac{\partial \theta}{\partial z}. \quad \dots\dots\dots\dots\dots\dots (12)$$

Integration of (10) gives

$$u = \frac{v_*}{k}\left\{ ln\frac{z+z_0}{z_0} + 2(\rho-1) - ln\frac{\rho(\rho+1)}{2}\right\}, \quad \dots\dots\dots\dots (13)$$

where

$$\rho = \left(\frac{\partial u}{\partial z}\right)_s \Big/ \left(\frac{\partial u}{\partial z}\right)_{ad} = \sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\gamma^2(z+z_0)^2}{\left(\frac{v_*}{k}\right)^2}}}$$

$$= u_{ad} + \frac{v_*}{k}\left\{2(\rho-1) - ln\frac{\rho(\rho+1)}{2}\right\}, \quad \dots\dots\dots\dots\dots\dots (14)$$

and

$$u_{ad}=\frac{v_*}{k}ln\frac{z+z_0}{z_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

denotes the value of u in the adiabatic atmosphere. Further they gave the eddy viscosity coefficient as

$$K=\frac{k(z+z_0)v_*}{\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\gamma^2z^2}{\left(\frac{v_*}{k}\right)^2}}}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

and the expression

$$\frac{u-u_{ad}}{\frac{v_*}{k}}=f(\rho)=2(\rho-1)-ln\frac{\rho(\rho+1)}{2} \quad \ldots\ldots\ldots\ldots\ldots (17)$$

represented graphicaly.

The wind velocity formula (13) is complicated and has a form which makes the treatment not easy, but it is shown at once that for larger values of stability one has, with increasing degree of accuracy

$$u\rightarrow\frac{\sqrt[4]{\gamma^2}}{\sqrt{\frac{v_*}{k}}}\sqrt{z}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

With the formula thus obtained, they tried to explain Heywood's data and showed a good agreement between the theory and experiment as is seen in fig. 3. Moreover they showed a greater part of Hellmann's experimental points lie between the two theoretical limits,i. e.

in the adiabatic atmosphere

$$\frac{u_{32}-u_{16}}{u_{16}-u_2}=\frac{ln\,32/16}{ln\,16/2}=0.33$$

in the most stable atmosphere

$$\frac{u_{32}-u_{16}}{u_{16}-u_2}=\frac{\sqrt{32}-\sqrt{16}}{\sqrt{16}-\sqrt{2}}=0.64$$

as is seen in fig. 3.

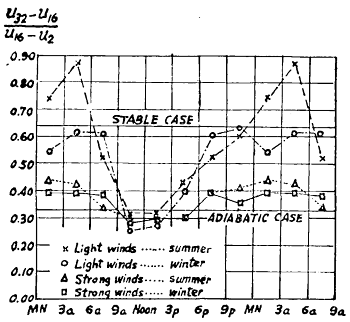

Fig. 3. Diurnal variation of the vertical wind distribution $(u_{32}-u_{16})/(u_{16}-u_2)$ for different wind velocities and seasons.

Rossby and Montgomery did not take into account the variation of lapse rate of temperature with height which is the common phenomenon in the lowest atmosphere. Later Sverdrup[42] put

$$\theta=\theta_0+b(z+z_0)^{\frac{1}{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

i. e.

$$\gamma^2=\frac{b\sigma g}{n\theta}(z+z_0)^{\frac{1-n}{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

and obtained

$$u = u_{ad} + \frac{n}{n+1} \frac{2v_*}{k} \left\{ 2(\rho'-1) - ln\frac{\rho'(\rho'+1)}{2} \right\} \quad \cdots\cdots\cdots (21)$$

and

$$K = \frac{k(z+z_0)v_*}{\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4\alpha^2(z+z_0)^{\frac{n+1}{2}}}{\left(\frac{v_*}{k}\right)^2}}}} \quad \cdots\cdots\cdots\cdots\cdots (22)$$

where u_{ad} is the same as in (15), and ρ' is obtained by substituting (20) in (14). From experiment over the snow field Sverdrup found

$$\sigma = 11.$$

Rossby and Montgomery's theory seemed to receive general recognition at the time, but after that its validity came into question. For instance, as to the fundamental assumption (Ⅱ) Brunt wrote in his PHYSICAL AND DYNAMICAL METEOROLOGY that it was "doubtful assumption," and really this is contrary to our experience that in the stable atmosphere layers of air become easy to glide over each other and make shearing stress smaller. As to the fundamental assumption (Ⅰ) the author[48] recently described as "groundless," because a correct energy equation must be the equation of energy dissipation which is deduced from the equation of motion. So it is natural that σ should vary with stability as recently found out by Deacon[4], i. e. for extremely unstable condition $\sigma \fallingdotseq 2$ and for markedly stable condition $\sigma \fallingdotseq 20$.

Moreover, Rossby and Montgomery and also Sverdrup considered that the wind profile was the same in the adiabatic and unstable atmosphere which was supported by Sverdrup's experiment of a few observational heights, but it is now widely accepted that it is not the case. (In the later paper Sverdrup[44] seems to consider that the profile is different in the adiabatic and unstable atmosphere.)

(f) On the controversy between Sutton and Sverdrup

In 1936 Sutton[33] remarked that if the wind profile was represented by

$$\frac{u}{u_1} = ln\left(\frac{az}{z_1}+1\right) \Big/ ln(a+1) \quad \cdots\cdots\cdots\cdots\cdots\cdots (23)$$

where u_1 is the value of u at the height z_1, the parameter a provided a very sensitive indicator of turbulence——i. e. a increased very rapidly on the lapse side. Against him Sverdrup[43], based upon Rossby and Montgomery's theory, wrote that (23) was valid only in the adiabatic atmosphere and enormous range of the values of a which Sutton obtained from temperature and wind observations at two levels showed not that a was a sensitive indicator of turbulence, but that the logarithmic law failed to hold good when the temperature gradient differed from the adiabatic.

In the subsequent paper Sutton[39] criticized Sverdrup's theory and wrote "like all modern mathematical studies on atmospheric turbulence, this analisis is not exact, and depends in the first instance on certain assumptions. An appeal to experiment is therefore essential". He then showed that Best's observation was represented more cosely by logarithmic than power law, and interpreted z_0 $\left(=\frac{z_1}{a}\right)$, as defined by Prandtl, to be possible of being a function of stability. But Sverdrup[44] still maintained the validity of tne theory, and, contrary to Sutton's opinion, tried to show that the Best's data can be interpreted differently and lend strong support to the statements that the roughness length was a characteristic physical constant and that the influence of stability can be expressed by means of another constant σ.

Since the nthis subject seems to have been discussed by none else but Takeda[45] who, recognizing the defect of the theory and observing that his own wind profile experiment could be represented by logarithmic law, supported Sutton. (Recently Halstead[11] describes that "since there were no observations of sufficient accuracy to provide sound judgment, controversies such as that between Sverdrup and Sutton were never clearly resolved.") And about fifteen years have elapsed since Sutton first published his paper, and meanwhile precise experimental data have increased, and Sutton[40] himself describes recently that if the greater height than 2 or 3 m. above the ground is considered, the evidence that shows the failure of logarithmic law is being accumulated.

As to the theory also some developments have been made. So Kawahara[20] tried to apply Karman's[19] energy equations to the lowest atmosphere and Lettau[27] attempted to improve the Rossby and Montgomery's theory. But as Kawahara uses some assumptions, such as the constancy of turbulent energy $(\overline{u'^2}+\overline{v'^2}+\overline{w'^2})$ with height, which need more verifications in the atmosphere, and as Lettau's treatment contains some ambiguities, both theories seem still not to be sound.

Lettau tries to improve the Rossby and Montgomery's energy equation (I) by replacing it with an acceleration equation, which seems to be incapable of being deduced from the equation of motion just as the energy equation is incapable of being deduced from the equation of energy dissipation. But the author considers if the Lettau's acceleration equation is true it should be that which can be deduced from the equation of motion.

Recently Halstead[11] considers that τ is not constant with height in the surface layer and putting $\tau=\tau_0+bz$ (where τ_0 is the value of τ at the surface and $b \gtreqqless 0$ for $\begin{array}{l}\text{stable}\\ \text{adiabatic}\\ \text{unstable}\end{array}$ atmosphere) gets some conclusions which can explain actual results. But to consider τ variable in the surface layer produces other difficulty, for instance, Halstead's assumption makes $\frac{\partial \tau}{\partial z}>0$ for stable atmosphere, i. e. the air is accelerated at night (and vice versa in the daytime) which is contrary to the case.

(g) Paeschke's[38] measurements

A set of measurements on wind-, temperature-, and humidity profile was made by Paeschke, who found that the exponent n varied from 3.0 to 5.0, became smaller as the roughness of the surface decreased, and, on the other hand, represented wind profile by a logarithmic law

$$u=\frac{v_*}{k}ln\frac{z-d}{z_0}, \quad \cdots\cdots(24)$$

where d is a height introduced to adjust to various roughness, and z_0 has the same meaning as before. Paeschke put

$$z_0=\frac{k}{7.35}, \quad \cdots\cdots(25)$$

where k is the height of unevenness of the surface or the height of the overgrowth. From direct measurements of stalk-length d, and u and z he showed that $k=d$ though there was a fair scattering of values.

Paeschke made some analysis concerning with stability but not with logarithmic profile, so it is of little interest to the author.

It may be added here concerning the form of $ln(z\pm d)$. There still remain some uncertainties as to the form of the logarithmic formula in the adiabatic case. For Rossby and Montgomery[32], Sverdrup[42] and Sutton[33] adopted the type

$$u=aln\frac{z+z_0}{z_0}, \quad \cdots\cdots(26)$$

while Paeschke[28] and Thornthwaite and Holzman[52] used

$$u=aln\frac{z-d}{z_0}. \quad \cdots\cdots(27)$$

So we are at a loss which to select.

If there is a surface, whose roughness shall be characterized by a quantity h, and if the wind profile over the surface is represented by

$$u=aln\frac{z}{z_0}, \quad \cdots\cdots(28)$$

we can assume h is proportional to z_0, i. e.

$$h=\gamma z_0. \quad \cdots\cdots(29)$$

If the form and height are constant and only the density of the roughness varies, we will probably obtain profile that shows reference surface elevated or lowered

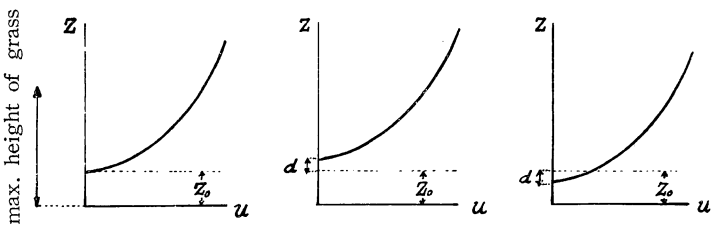

Fig. 4. Wind profile and density of element.

A: Density of roughness: medium $u\sim ln\frac{z}{z_0}$

B: Density of roughness: large $u\sim ln\frac{z-d}{z_0}$

C: Density of roughness: small $u\sim ln\frac{z+d}{z}$

as is shown in fig. 4. If the form of the curve remains the same, we obtain for B and C of the figure

$$u = a\,ln\frac{z \mp d}{z_0}. \quad \ldots\ldots (30)$$

But it is doubtful whether the curve remains the same after the density varies. We may also obtain in that case

$$u = a\,ln\frac{z}{\zeta_0}, \quad \text{where} \quad \zeta_0 = z_0 \mp d. \quad \ldots\ldots (31)$$

If we assume

$$h = \gamma' \zeta_0, \quad \ldots\ldots (32)$$

γ' is, of course, different from γ. The experience of the author shows that over various natural surfaces u is proportional to lnz in the adiabatic case, hence that the formula (31) holds, and it is not necessary to use (26) or (27). So in view of the pure empirical nature of the formula the author considers it sufficient to adopt (28) and only when experiments (made near the roughness height) show some deviation from (28) we should use (30) in agreement with conclusion obtained by Deacon[4] that in conditions of neutral stability the logarithmic law can represent the profile between heights of 1 m. and 13 m. over the grass of various heights with great accuracy, provided that both z_0 and d are chosen independently to give the best fit.

(h) Takeda's[45] measurements

Nineteen measurements of wind- and temperature profile either in stable or unstable conditions were made by the author over a natural surface with not small and not uniform roughness (maximum height of the shrub reached about 1.5 m.). Obtained results at 4 heights up to 5 m. showed that the exponent n varied with height and stability from 0.5 to 4.0 (increasing with height and decreasing with stability), and that the logarithmic law was better fitted than the exponential law, and if z_0 was considered to vary according to stability, i. e. increasing with stability, the simple logarithmic formula

$$u = a\,ln\frac{z}{z_0} \quad \ldots\ldots (33)$$

fitted well in the limit of the experimental error. From the fact that the logarithmic law held also in the non-adiabatic atmosphere the author deduced for the stability dependence of K, v_*, and k as follows:

$$K_s = \frac{K}{1+\sigma R_i}, \quad \ldots\ldots (34)$$

$$v_{*s} = \frac{v_*}{\sqrt{1+\sigma R_i}}, \quad \ldots\ldots (35)$$

and

$$k_s = \frac{k}{\sqrt{1+\sigma R_i}}, \quad \ldots\ldots (36)$$

where R_i (Richardson's no.) $= \frac{g\partial\theta}{\theta\partial z} \Big/ \left(\frac{\partial u}{\partial z}\right)^2$, and suffix s is applied to denote quantities in the stratified atmosphere. But the deviation from the logarithmic law obtained recently makes the author adopt another formulae which will be described later.

(Ⅱ) Deviation from logarithmic law

(i) Thornthwaite and other's measurements

Though some deviations from logarithmic law were observed in Best's and Sverdrup's data they were not so powerful as to claim their existence, for in those days the problem of the profile being represented by logarithm or exponential itself was not settled and Sverdrup's[43] data were obtained over snowfield with only 3 heights of measurement. But in the wartime the existence of the deviation was being observed more certainly. So Thornthwaite and Halstead[51] from measurements of profile by 6 heights up to 20 ft. found the deviation and proposed a rather untractable combination of logarithmic and power terms

$$u=\left(\frac{lnz-lnz_0}{lna}\right)^{\frac{1}{p}}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

where the exponent p was expected "to vary between 2.0 with fully developed turbulence and some value less than 1.0 when turbulence reaches its smallest actual value." To quote Sheppard after Halstead[11]: "In this respect the most notable published profiles are those of Thornthwaite and Kaser (1943), taken over a flat field in Ohio at up to 12 levels between 0.5 ft. and 28 ft. The u, lnz curves for successive hours throughout the day show a marked progression of form, being concave to the u-axis between sunset and sunrise, that is during the period of temperature inversion, linear shortly after sunrise and before sunset when conditions are approximately dry-adiabatic and convex to the u-axis during the central daylight hours of superadiabatic lapse rate. Halstead (1943) has shown that the curvature of their profiles is intimately related with the temperature difference which was recorded between 2 ft. and 8 ft."

(j) Deacon's[4] measurement

One of the most systematic relations between the deviation and stability is given by Deacon recently, who arranged his data according to the mean Richardson's no. between the height 4 and 0.5 m., $J_{4:0.5}$. It is reproduced on fig. 5. It is clear from the figure that in unstable conditions u, $\log z$ distribution is convex to the u-axis and in stable conditions it has an opposite curvature, and the curvature itself is large

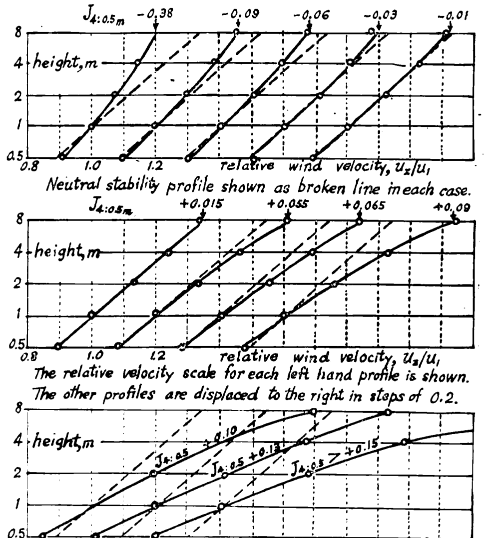

Fig. 5. Variation of wind distribution with stability.

as the difference from adiabaticity becomes large.
He found that the formula of the type

$$\frac{du}{dz}=az^{-\beta}, \qquad \text{.......................... (38)}$$

where
$\beta>1$　for unstable condition,
$\beta=1$　for neutral condition,
$\beta<1$　for stable condition,

represents the data quite closely, and gave the variation of β with stability in a figure.

The integration of (38) leads to a new profile

$$u=\frac{az_0^{1-\beta}}{1-\beta}\left\{\left(\frac{z}{z_0}\right)^{1-\beta}-1\right\}, \qquad \text{..................... (39)}$$

and if

$$a=\frac{v_*}{kz_0^{1-\beta}}, \qquad \text{.......................... (40)}$$

(39) becomes

$$u=\frac{v_*}{k(1-\beta)}\left\{\left(\frac{z}{z_0}\right)^{1-\beta}-1\right\}. \qquad \text{................... (41)}$$

Deacon states that, strictly speaking, the parameter β is not constant with height and the deviation of β from 1 increases to some extent with height. But the non-constancy of β with height seems to arise from the fact that Deacon makes z_0 constant with stability. Deacon, ignoring actual variation, considers z_0 is constant, because as the Richardson's no. varies almost linearly with height the effect of buoyancy must become very small in the layer near the earth's surface and results based on many observations show that the effect of temperature gradient upon velocity distribution decreases as the surface is approached as follows:

Ratio of wind	Percentage variation† of wind on the given stability range
8 : 4 m.	7.2
4 : 2 m.	5.5
2 : 1 m.	4.2
1 : 0.5m.	2.6

† stability range　from −0.1 to +0.06

But though the Richardson no. varies almost linearly with height, it seems dangerous to extrapolate this to the layer just above the roughness, and, moreover, it may be shown

$$\frac{d}{dz}\left[\left(\frac{u_{2z}}{u_z}\right)_2-\left(\frac{u_{2z}}{u_z}\right)_1\right]>0, \qquad \text{................... (42)}$$

where suffix 1 shows the unstable case and 2 the stable one, is valid in the height interval of z observed by Deacon (4 m.−0.5 m.) even though z_0 varies. For adopting Deacon's formula (39) we have

$$\frac{u_{2z}}{u_z}=\frac{(2z)^{1-\beta}-z_0^{1-\beta}}{z^{1-\beta}-z_0^{1-\beta}}. \qquad (43)$$

If β and z_0 is considered to vary with stability and (43) is substituted in the left-hand side of (42) we have

$$\frac{d}{dz}\left[\left(\frac{u_{2z}}{u_z}\right)_2-\left(\frac{u_{2z}}{u_z}\right)_1\right]=\left(\frac{(1-\beta)z^{-\beta}z_0^{1-\beta}(1-2^{1-\beta})}{(z^{1-\beta}-z_0^{1-\beta})^2}\right)_2-\left(\frac{(1-\beta)z^{-\beta}z_0^{1-\beta}(1-2^{1-\beta})}{(z^{1-\beta}-z_0^{1-\beta})^2}\right)_1. \qquad (44)$$

From Deacon's results we have

case 1: $J_{4:0.5}=-0.1,$ $\beta_1=1.10,$ $z_{01}=0.45$ cm.,

case 2: $J_{4:0.5}=+0.06,$ $\beta_2=0.92,$ $z_{02}=0.10$ cm.

If these values are substituted in (44) we can obtain the domain of z satisfying (42), and get

$$z>0.88 \text{ cm}. \qquad (45)$$

The lowest height adopted by Deacon was 50 cm., and hence Deacon's suggestion, that the effect of stability on wind velocity becomes smaller as the earth's surface is approached, will not prove the constancy of z_0.

The form of a will be examined next. Deacon expands the right-hand side of the equation (39) and comparing it with the equation which is valid in the adiabatic condition he obtains (40). But it is easily seen that a may have a form

$$a=\frac{v_*}{k\zeta^{1-\beta}}, \qquad (46)$$

where ζ denotes an arbitrary length in place of z_0. This is absurd, so we should not expand (39) and put $\beta=1$ carelessly. A correct form of a will be obtained from (39) and

$$\frac{u}{U}=\frac{z^{1-\beta}-z_0^{1-\beta}}{z_1^{1-\beta}-z_0^{1-\beta}}, \qquad (47)$$

where U denotes the velocity at $z=z_1$, as follows:

$$a=\frac{U(1-\beta)}{z_1^{1-\beta}-z_0^{1-\beta}}. \qquad (48)$$

In the case of the adiabatic condition, $\beta=1$, and $U=\frac{v_*}{k}ln\frac{z}{z_1}$, so a becomes

$$a=\frac{v_*}{k}. \qquad (49)$$

But we should not expect that this relation holds also in the nonadiabatic atmosphere. Deacon then obtains a formula of eddy viscosiy as follows:

$$K=kv_*z_0\left(\frac{z}{z_0}\right)^\beta, \qquad (50)$$

which should be corrected to

$$K=\frac{v_*^2(z_1^{1-\beta}-z_0^{1-\beta})}{U(1-\beta)}z^{\beta}. \quad \cdots\cdots\cdots\cdots(51)$$

(k) Pasquill's[29] measurements

The most interesting simultaneous measurements of wind velocity, temperature and humidity are made by Pasquill at 6 heights up to 2 m. It is extremely remarkable that wind velocity, temperature and humidity show the same deviation from the logarithmic profile as is seen from fig. 6. From the figure it seems probable that wind, temperature and humidity distribution in the lowest layer are dtermined by the same agency——turbulence. But the temperature distribution is shown by the author to have a greater deviation than the other two. This may be due to the effect of buoyancy or radiation, but we shall need more experiment to determine this effect.

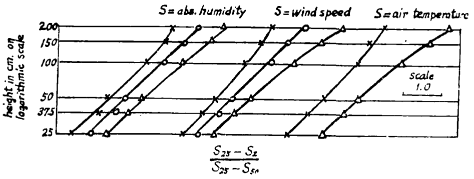

Fig. 6. Vertical profiles of absolute humidity, wind speed and air temperature above short grass.

It may be added that Pasquill, trying to verify the equality of K and K_v (coefficient of eddy diffusivity for water vapour) experimentally, used $K\Big/z^2\frac{\partial u}{\partial z}=k^2\left(\frac{z}{z_0}\right)^{2\beta-2}$ (which is easily deduced from (41) and (50)) and assuming that z_0 is constant with stability calculated this value from his wind observations and compared it with experimentally obtained $K_v\Big/z^2\frac{\partial u}{\partial z}$. He could conclude that $K=K_v$ in the unstable as well as in the adiabatic cases, but could not conclude that $K=K_v$ in the stable case as the values of $K\Big/z^2\frac{\partial u}{\partial z}$ became too small. If, on the contrary, (39) and the corrected formulae (48) and (51) are used, we can obtain

$$K\Big/z\frac{\partial u}{\partial z}=\frac{v_*^2(z_1^{1-\beta}-z_0^{1-\beta})}{U^2(1-\beta)^2}z^{2\beta-2}. \quad \cdots\cdots\cdots\cdots(52)$$

Assuming the value of $v_*/U=1/14.4$, which is obtained in the adiabatic case, is constant also for the non-adiabatic case, we can evaluate $K\Big/z^2\frac{\partial u}{\partial z}$ as shown in the table. It is seen that the values are in good agreement for all cases, though they are somewhat smaller especially in the unstable case. But it seems to the author that the experimental verification of the equality of K and K_v is given for all conditions of stability of the atmosphere.

Table

Richardson's number		−0.125	−0.10	−0.05	0	+0.05	+0.10	+0.125
Pasquill's value	$K\Big/z^2\frac{\partial u}{\partial z}$	0.42	0.37	0.26	—	0.06	0.016	0.005
	$K_v\Big/z^2\frac{\partial u}{\partial z}$	0.39	0.34	0.24	—	0.11	0.07	0.06
Takeda's value	$K\Big/z^2\frac{\partial u}{\partial z}$	0.25	0.24	0.22	0.16	0.10	0.06	0.04

(Ⅲ) Summary about the profile and search for the most appropriate formula.

Now we have come to the stage of summarizing profiles and searching for the most appropriate formula.

(a) If the exponential law

$$u = az^{\frac{1}{n}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (53)$$

is to be applied:

n varies with z, inceasing as z becomes large .... Hellmann[12),13),14),15),16)], Best[1)], Takeda[45)],

n varies with roughness, increasing as roughness height becomes small .... Paeschke[23)],

n varies with stability, increasing as the layer becomes less stable.. Best[1)], Takeda[45)].

Values of n are found to vary from 0.5 to 5 in the atmosphere.

(b) If the logarithmic law

$$u = a \ln z + b \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (54)$$

is to be applied:

Many observations are represented by this law well and, above all, almost exactly for the adiabatic case.

When the roughness of the surface has some particular feature instead of (54) for the adiabatic case

$u = a \ln(z + z_0) + b$ ....Rossby and Montgomery[32)], Sverdrup[42)], Sutton[38)],..(55)

or

$u = a \ln(z - d) + b$ ....Best[1)], Paeschke[23)],..(56)

will be better fitted.

If (54) is written as

$$u = a \ln \frac{z}{z_0}, \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (57)$$

z_0 varies with stability, increasing as the air layer becomes stable...........Thornthwaite and Holzman[53)], Sutton[41)], Takeda[45)].

(c) For the more precise measurements, deviation from logarithmic law has been observed. Some formulae are proposed in order to give the best fit, i. e.

theoretically:

$u = a \ln(z + z_0) + F(z)$ ..Rossby and Montgomery[32)], Sverdrup[42)], Lettau[27)],..(58)

$u = a \ln z + bz + c$ ....Kawahara[20)], Halstead[11)],..(59)

empirically:

$u = \left(a \ln \frac{z}{z_0} \right)^{\frac{1}{p}}$ ....Thornthwaite and Halstead[51)],..(60)

$u = a \left\{ \left(\frac{z}{z_0} \right)^{1-\beta} - 1 \right\}$ ....Deacon[4)],..(61)

$$u = a \ln z + b \ln^2 z + c \qquad \text{Takeda}^{47)}. \tag{62}$$

The author has shown that the formula of the type (61) or (62) is better fitted to Best's and Pasquill's data than that of the type (59).

It should be mentioned here about the variability of z_0 with stability. As described above z_0 is an integration constant which adjusts itself that at the surface of the roughness u equals to the actual velocity. If z_0 is regarded as an unknown parameter and obtained from actual profile, it will be found that it does vary with stability. But there is a group of researchers who consider that z_0 is a physically definable height and is not influenced by stability. To this group belong Rossby and Montgomery[33], Sverdrup[42], Kawahara[20] and Deacon[4]. On the other hand Sutton[41], Thornthwaite and Holzman[53], and Takeda[45], consider that z_0 may vary with stability, i. e. increasing as the air layer becomes stable (in a recent paper the author[46] has shown that z_0 decreases with stability so long as the formula representing the deviation from the logarithmic law, such as Deacon's generalized exponential formula or Takeda's generalized logarithmic formula, is adopted).

In this connection it should be stated that the result obtained by Lettau[27] is very remarkable. For Lettau distingishes the physically definable roughness height z_0, which does not vary with stability, from the integration constant "z_0", which is hitherto considered to vary in actual cases, and deduced a relation

$$``z_0" \doteqdot z\left(\frac{z_0}{Yz}\right)^X, \tag{63}$$

where X and Y denote some functions of stability. This relation (63) implies that "z_0" is a function of z, so we shall obtain different values of "z_0" if different reference height is used. But there seems to exist no experiment now that can ascertain this expression.

After all, though the theoretical ground may be wanting, Deacon's formula with variable z_0, i. e. decreasing with stability, seems to the author to be the most simple and the best fitted to experiments at present, and it will be adopted as the starting point of the following analysis.

§ 2. Mixing length and eddy viscocity

Having decided to adopt Deacon's formula

$$u = \frac{a z_0^{1-\beta}}{1-\beta}\left\{\left(\frac{z}{z_0}\right)^{1-\beta} - 1\right\}, \tag{64}$$

or

$$\frac{\partial u}{\partial z} = a z^{-\beta}, \tag{65}$$

for

$\beta > 1$ unstable case,

$\beta = 1$ adiabatic case,

$\beta < 1$ stable case,

where z_0, a and β are parameters depending only on the stability, as our starting point, we now proceed to determine the mixing length and the eddy viscosity in relation to stability. The widely accepted formulae[31] between the mixing length, the eddy viscocity and the frictional velocity are

$$K\frac{\partial u}{\partial z}=v_{*}^{2}, \qquad (66)$$

and

$$l\frac{\partial u}{\partial z}=v_{*}. \qquad (67)$$

If we assume that these formulae hold also in the non-adiabatic atmosphere and comparing (65) and (67) we can obtain

$$\frac{v_{*}}{l}=az^{-\beta}. \qquad (68)$$

v_{*} shall be assumed here to be independent of z with English researchers[3)'41)] though it may depend on stability. We have, then,

$$l\sim z^{\beta}, \qquad (69)$$

or

$$l=A(\beta)z^{\beta}, \qquad (70)$$

where A is a proportionality factor depending only on stability. As β is shown to be determined only by the Richardson's number at a certain height, it is convenient to adopt β as an index representing stability of the atmosphere because it does not contain z. But in the adiabatic atmosphere (70) must be reduced to

$$l=kz, \quad \text{where} \quad k=0.4, \qquad (71)$$

so we can put instead of (70)

$$l=kh(\beta)z^{\beta}. \qquad (72)$$

With this $h(\beta)$ the velocity distribution becomes from (64) and (68)

$$u=\frac{v_{*}}{k(1-\beta)h}(z^{1-\beta}-z_0^{1-\beta}), \qquad \left(a=\frac{v_{*}}{kh}\right) \qquad (73)$$

and eddy viscocity from (66) and (67)

$$K=kv_{*}hz^{\beta}. \qquad (74)$$

It may be remarked here that the Deacon's velocity and eddy viscosity formula (41) and (50) agree with (73) and (74) respectiyely if we put

$$h=z_0^{1-\beta}, \qquad (75)$$

but there is no reason to adopt (75). But if we put $u=U$ when $z=z_1$, we have from (73)

$$\frac{u}{U}=\frac{z^{1-\beta}-z_0^{1-\beta}}{z_1^{1-\beta}-z_0^{1-\beta}}, \qquad (76)$$

and from (73) and (76)

$$h=\frac{v_{*}}{U}\cdot\frac{z^{1-\beta}-z_0^{1-\beta}}{1-\beta}, \qquad (77)$$

and with (74) we have

$$K=\frac{v_*^2(z_1^{1-\beta}-z_0^{1-\beta})}{U(1-\beta)}z\beta, \qquad \cdots\cdots(78)$$

which is already obtained above.

The evaluation of $h(\beta)$ is not simple for lack of appropriate data, but we can be able to make use of Pasquill's[23] results for this purpose. It is already remarked that the author showed $K=K_v$ (eddy diffusivity for water vapour) with Pasquill's data assuming that v_*/U is independent of stability (or varies only a little in a degree which does not make the order of magnitude change). Now from (73) and (74) we have

$$K\Big/ z^2\frac{\partial u}{\partial z}=k^2h^2z^{2\beta-2}. \qquad \cdots\cdots(79)$$

So if we assume $K=K_v$ we can obtain values of h equating (79) with Pasquill's $K_v\Big/ z^2\frac{\partial u}{\partial z}$. In fig. 7 is shown the variation of h with Richardson's number thus obtained together with that of β. It is interesting to note that the values of h and β are fairly in agreement except in the unstable atmosphere where h becomes larger than β. In the same figure also two sorts of values of $z_0^{1-\beta}$ are plotted, one in the case z_0 does not vary with stability and equals to its adiabatic value $z_0=0.0025$ m. and with values of β as given by Pasquill, and the other in the case z_0 varies with stability and with values of β given already in the same figure. It is seen that the former agrees fairly well with h except in the stable atmosphere where $z_0^{1-\beta}$ becomes smaller than h as expected, and the latter's variation is very large and becomes negative near $R_i=0.11$.

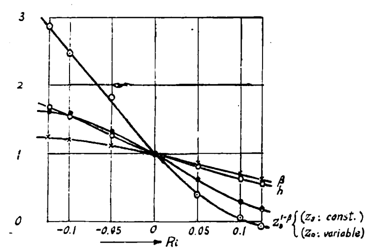

Fig. 7. Relation of $z_0^{1-\beta}$, β and h with Ri.

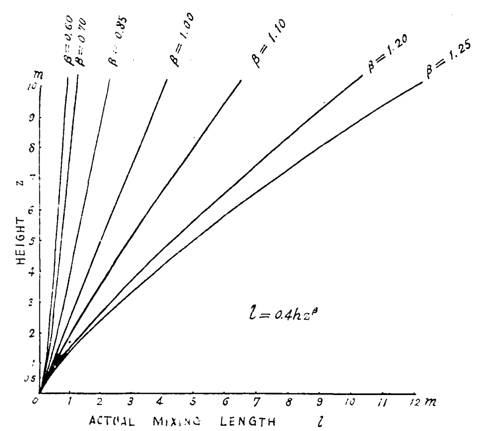

Fig. 8. Variation of actual mixing length with height.

In fig. 8 is shown variation of the actual mixing length (72) with height for various values of β. This may be compared with Lettau's[27] result (fig. 3 in Lettau's paper), and it is seen the general tendencies are the same but our curves show somewhat less curvature. The figure is still more worthy of notice because it shows the variation of K/v_* with height. As v_* is assumed here to be independent of height though it may depend on stability, fig. 8 shows also the variation of K with height. Sverdrup[42] gave the variation of K for stable and adiabatic condition, whose tendencies are in agreement with fig. 8, but did not give for stable condition. Fig. 8 shows that the tendency of variation of K with height for unstable condition is contrary to that for stable condition, i. e. convex to z-axis contrary to concave.

Now we examine the variation of v_{*}/U with stability with regard to the Pasquill's data under the assumption of $K=K_r$. From the evaluated h and a/U values of v_{*}/U are obtained and plotted in fig. 9. Though v_{*}/U varies only a little with stability (only about 20% or less) as anticipated, it is a remarkable and strange feature that it has a minimum at $R_i=0$. Is Pasquill's experiment not open to any criticism? Did some factor which had not been considered make K_r, hence v_{*}/U, excessively large in the unstable atmosphere? Pasquill discussed the movement of water in his evaporimeters and with different soil conditions, and though his experiment seems to have been without objection yet we can not help considering that this is one of the most important sections of the experiment and some more repetition is desirable*.

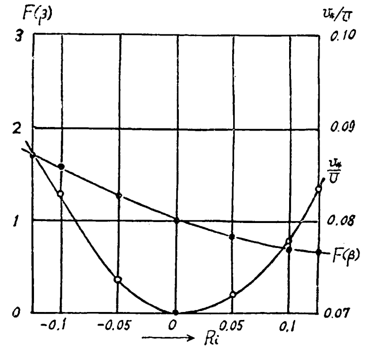

Fig. 9. Relation of v_{*}/U and $F(\beta)$ with Ri.

§ 3. On Thornthwaite's evaporation formula

To determine actual transpiration or evaporation from large surfaces Thornthwaite and Holzman[52] used an evaporation formula as follows:

$$E=\frac{k^2\rho(q_1-q_2)(u_2-u_1)}{\left(ln\frac{z_2}{z_1}\right)^2}, \qquad \cdots\cdots(80)$$

where E denotes the rate of evaporation, ρ density of the air, q_1 and q_2, and u_1 and u_2 are moisture concentration and wind velocity at z_1 and z_2 respectively. This formula, deduced for an adiabatic atmosphere, must be applied by a correction for a precise evaluation of evaporation in cases of stability. But they did not give the correction.**

Recently Pasquill[29] gave the formula as follows:

* During the preparation of this paper the author read Pasquill's subsequent paper (Quart. Journ. Roy. Met. Soc., 76, (1950), 287—301) in which additional experimental results were published, but it is regrettable that Pasquill's results are mainly concerned with adiabatic condition (only two cases are for unstable condition). It is hoped that he can publish experimental results for all conditions of the atmosphere.

** After this paper was written the auther read Holtzman's paper ("The Influence of Stability on Evaporation," Annuls of New York Academy of Science, Vol. XLIV, art. 1, 1943, pp. 13—18.), in which the effect of stability on evaporation was already considered. But as Holtzman's formula is not connected with a simple wind velocity profile, it seems still of value to give a derivation as described here which is connected with the latest wind velocity formula such as Deacon's.

$$E=\frac{(1-\beta)^2k^2\rho z_0^{2(1-\beta)}(q_1-q_2)(u_2-u_1)}{(z_2^{1-\beta}-z_1^{1-\beta})^2}, \quad \ldots\ldots\ldots\ldots\ldots (81)$$

which is easily deduced from (41) and (50). But as (41) and (50) are not correct we must deduce an alternative form. This can be done at once by replacing $z_0^{1-\beta}$ by h and be written

$$E=\frac{k^2\rho(q_1-q_2)(u_2-u_1)}{\left(ln\frac{z_2}{z_1}\right)^2}.F(\beta), \quad \ldots\ldots\ldots\ldots\ldots (82)$$

where

$$F(\beta)=\left(\frac{(1-\beta)h\,ln\frac{z_2}{z_1}}{z_2^{1-\beta}-z_1^{1-\beta}}\right)^2. \quad \ldots\ldots\ldots\ldots\ldots (83)$$

In another paper Pasquill[30] describes that neglect of the influence of thermal stratification, however, introduces systematic error in the form of an underestimation in unstable conditions and an overstimation in stable conditions, to an extent which is systematically related to the degree of instability or stability as specified by the Richardson's number. He can estimate from his observations that the errors are mainly within 10 per cent, but for only a few observations in the daytime unstable conditions the error becomes greater than this but less than 20 per cent. For a number of the nocturnal observations the Richardson's number can not be estimated with any confidence, due to lightness of the wind, and it is possible that overestimations in excess of 10 per cent will apply, but these are invariably associated with very low absolute magnitudes of vapour transport.

From values of h given above we can evaluate $F(\beta)$——the departure of actual values from adiabaticity——from (82), and obtained $F(\beta)$ is shown in fig. 9. It is seen that $F(\beta)$ is larger than 1 in the unstable atmosphere, i. e. the systematic error is in the form of the underestimation, and in the stable atmosphere it is overestimation as expected. But the departure from 1 is somewhat larger than the Pasquill's observations described just above, and amount to more than 50% for $R_i=-0.1$ and 30% for $R_i=+0.1$. It is clear from (83) that the value of $F(\beta)$ becomes large as h increases. The somewhat large values of $F(\beta)$ seem to associate to too large values of h in the unstable atmosphere, but in this respect we shall need more experiments.

It may be remarked that the values of $F(\beta)$ is here obtained from Pasquill's evaporation experiment. But the Thornthwaite's formula (80) or corrected formula (82) is primarily that of obtaining evaporation. So at present we can not obtain the exact value of evaporation from wind and moisture concentration of the air alone without knowing h. But as h essentially does not depend upon moisture, we shall be able to find h from other experiments in future, for instance, from measurements of τ_0 or temperature and radiation. Till then the application of Thornthwaite's method of obtaining evaporation from large areas in the non-adiabatic atmosphere appears to be postponed.

§ 4. **Summary.**

(1) It is shown that there are three methods of approach to the problem of turbulence in the lowest atmosphere, i. e. (a) theoretical, (b) empirical, from the mean wind and (c) empirical, from the fluctuation of wind.

(2) In order to adopt the second method of approach the author reviews principal works hitherto done concerning the vertical distribution of wind velocity, and gives some criticisms not only about experimental but also about theoretical works.

(3) It is remarked that it is sufficient to adopt the wind profile

$$u \sim lnz$$

in the adiabatic atmosphere for most cases and only when experiments (made near the roughness height) show some deviation from the profile we should use

$$u \sim ln(z \pm d).$$

(4) It is shown that the integration constant z_0, which adjusts itself that at the surface of the roughness u equals to the actual velocity, varies with stability; and that when the simple logarithmic formula

$$u = a ln \frac{z}{z_0}$$

is adopted, z_0 increases with stability, but that when the formula representing the deviation from the logarithmic law is adopted, z_0 decreases with stability, and probably this will be the case.

(5) It is shown that Deacon has interpreted the form of a in his newly deduced formula and that of eddy viscocity K erroneously and corrected forms are presented.

(6) The corrected form of eddy viscocity is shown to enable to prove experimentally the equality of $K = K_v$ (eddy diffusivity for water vapour), which was incapable for Pasquill, who used the Deacon's incorrect form.

(7) Deacon's velocity profile with variable z_0, i. e. decreasing with stability,

$$u = \frac{a}{1-\beta}(z^{1-\beta} - z_0^{1-\beta})$$

is considered to be the most simple and the best fitted to experiments at present, and adopted as the basis for the subsequent analysis.

(8) Making use of the Deacon's profile, the expression for the mixing length and the eddy viscosity are obtained as

$$l = 0.4h(\beta)z^{\beta},$$

and

$$K = 0.4v_{*}h(\beta)z^{\beta},$$

and the parameter h, which is considered to depend only on stability, is evaluated from Pasquill's experiments.

(9) The variation of v_{*}/U with stability is exmined with regard to the Pasquill's data and a strange feature is obtained that it has a minimum at

$R_i=0$. It is desirable that more experiments of this sort will be made.

(10) It is shown that the parameter $h(\beta)$ defined above appears in Thornthwaite's evaporation formula, and remarked that the application of Thornthwaite's method of obtaining the exact evaporation from large areas in the non-adiabatic atmosphere appears to be postponed till $h(\beta)$ will be got from other measurements than that of evaporation, i. e. such as that of τ_0 or temperature and radiation.

PART II Irregular or turbulent component of wind

§ 1. Introduction

The variation of the turbulent component of wind with height and stability seems to have been investigated only in a few cases. Scrase[35], Giblett[10] and Best[1] published some data about turbulent component of wind and above all Best showed that $\bar{g}\left(=100\frac{\overline{|u'|}}{\bar{u}}\right.$, $\bar{u}$ being mean wind velocity and $\overline{|u'|}$ being mean of the absolute value of velocity fluctuation) decreased very slowly with height in the surface layer up to 2 m., and gustiness obtained from records of bidirectional vanes decreased as the temperature gradient changed from lapse to inversion. Recently Frankenberger[9] has succeeded to obtain the stability dependence of eddy viscosity and turbulent stress from measurements of wind fluctuations and Shiotani[37] published results which showed vertical distribution of certain turbulent characteristics. In the following the author will also give results of measurements made several years ago which, though not necessarily precise, seem to show some interesting features.

§ 2. Method of observation

Observations were made on an abandoned field on the NNW-slope of Mt. Akagi, Gumma Prefecture. The field which at the time being used for army exercises now and then had an area of about 1 km^2. and the mean inclination was 4° down to NNW. The overgrowth was very irregular: there were low grasses as well as high grasses. Also shrubs as tall as 1 m. high were within 20 or 30 meters from the measuring spot. In short, the roughness of the field was large but the density of the roughness element was small.

Measurement of wind velocity and direction was made by the anemoscope with vane, now generally used in our country in field. From a preliminary experiment the instrument was found to show the wind direction at the wind velocity of about 0.7—0.8 m./sec., but the sensitivity for wind velocity was better and the anemoscope set in action at about 0.5 m./sec. Measuring heights were 5 m., 2 m., 1 m. and 0.5 m. from the ground, the measurement at 5 m. height being made on a simple wooden stand of about 3.5 m. high. To make a measurement four observers were necessary, who measured mean wind direction and velocity at each height respectively in a time interval of 10 sec. after the anouncement of a time keeper. The measurement lasted 20 min. each time.

The adoption of the time interval of 10 sec. was due to the time lag of the instrument. The preliminary experiment showed that the revolution rate of the anemoscope became to its half value in about 4 or 5 sec. when the air stream was suddenly intercepted, so it seemed incapable to reduce the time interval any more. To analyse the results statistically we wanted some hundred samples, so the 20 minutes' measuring time was adopted, thus giving 120 samples in each case. Moreover 20 minutes seemed to be the largest time interval to be selected easily in the time of the day in which remarkable weather changes did not occur.

Thus 12 measurements were made from 30 th Nov. to 5 th Dec. 1943, but those during which wind ceased or suddenly grew strong, i. e. those which could not be considered as being made in a stationary condition, were rejected and 8 measurements, all being made in fair weather, were obtained, in which 4 were made in the daytime and the remaining 4 in the evening. The vertical temperature distribution was also measured at the same time in each case, but these data were regrettably lost at the time of confusion after the war, so the degree of the stability remained unknown. But as the evening measurements were made after the sun had set in the mountain and the katabatic wind prevailed, it is obvious that they were made in the stable condition of the atmosphere.

§ 3. Results of the experiment

(Ⅰ) Mean wind velocity

The vertical distribution of mean wind velocity V are plotted in fig. 10 and 11, the former being the logarithmic representation of the height z of the latter. As the problem of the vertical distribution of mean wind velocity is treated fully in Part Ⅰ, we will not consider them in detail but only give a remark that they can be well represented by the logarithmic law, and besides some deviations from the law just in the same direction as described above are shown in the figure.

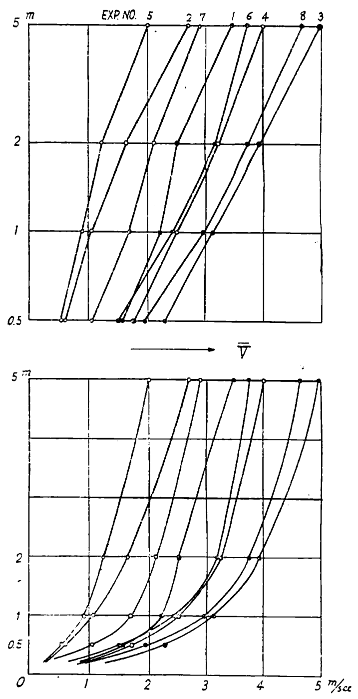

Fig. 10 and 11. Vertical distribution of mean wind velocity.

(Ⅱ) Energy and intensity of turbulence

From each observed instantaneous velocity V (=mean wind velocity in each 10 sec.) the turbulent component V' is obtained by the subtraction of $\overline{V}$ (=mean velocity in 20 min.), and $\overline{V'^2}$ and $\sqrt{\overline{V'^2}}/\overline{V}$ are calculated and showed in figures. Figs. 12 and 14 are the

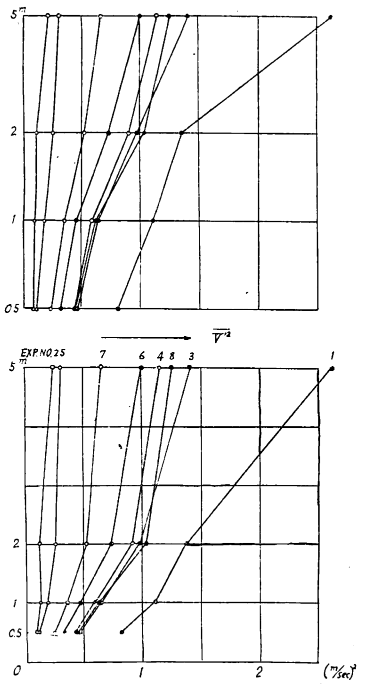

Fig. 12 and 13. Vertical distribution of energy of turbulence.

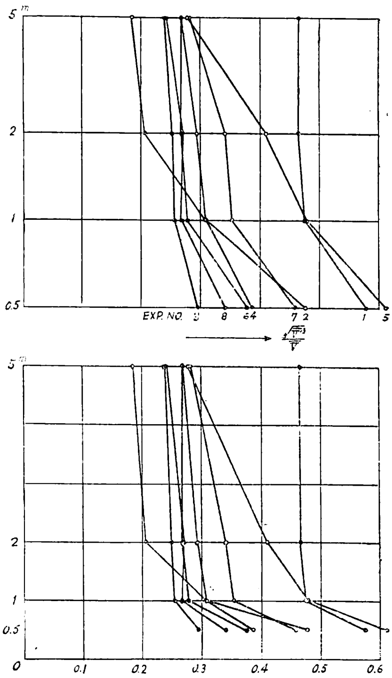

Fig. 14 and 15. Vertical distribution of intensity of turbulence.

logarithmic representation of the height z of the figs. 13 and 15 respectively as in the case of the figs. 10 and 11.

From figs. 12 and 13 it is clear that $\overline{V'^2}$ increases with height z, the rate of increase being somewhat larger than lnz. The stratification of the air layer may have some effect on the rate of increase but it is difficult to find out any law from our experiment except that each value of $\overline{V'^2}$ itself becomes smaller with stability. Best[1] found experimentally

$$|\overline{V}|=0.15\ lg(z-1)+\text{const},$$

which agrees with our results in the sense that $\overline{V'^2}$ increases more than with lnz.

The relative intensity of turbulence $\sqrt{\overline{V'^2}}\,/\,\overline{V}$, on the contrary, decreases with height as is seen from figs. 14 and 15. The decrease is very steep in the layer near the ground, i. e. in the layer 0.5 m.—1 m., but becomes slight or the intensity is almost constant in the layer above 2 m. Best states that $\sqrt{\overline{V'^2}}\,/\,\overline{V}$ is constant with height, though his results show a slight decrease.

The fact that $\overline{V'^2}$ increases with height in the surface layer makes existing theories, e. g. those of Rossby and Montgomery[32] and Kawahara[30], not correct because they assume the turbulent energy constant. Rossby's[33] previous theory, however, explains the fact, for he deduced from Richardson's energy equation

$$z=\frac{1}{\sqrt{C}}\cdot\frac{1}{\left(\frac{2\psi}{3C}\right)^{\frac{2}{3}}}\int_0^{x_1}\frac{xdx}{\sqrt{1-x^3}}, \quad \ldots\ldots(1)$$

where $x_1=\left(\frac{2\psi}{3C}\right)^{\frac{1}{3}}E$, E being the turbulent energy, and C is a constant, assuming $\psi=\left(\frac{\partial u}{\partial z}\right)^2-\frac{g}{T}\left(\frac{\partial T}{\partial z}+\Gamma\right)$ is constant with height. It is clear from (1) that $E=0$ for $z=0$. But the recent developments in turbulence make the author discontented with theories which do not take into account spectal considerations, and so he will not enter into details now until some more progress be made.

(Ⅲ) Frequency distribution of the variation of wind velocity

The frequency distribution of V' is obtained by enumerating number of occurrences of V' in each velocity intervals of 0.5 m./sec. around the mean, and is shown in fig. 16 in histogram. It is clear from the figure that, though there are some scattering, the variance is smaller as z becomes small, and as the air layer becomes stable.

The theoretical distribution was treated by Hesselberg and Björkdal[17], who obtained for the frequency distribution function (or probability density function) of V in the case of the 2-dimensionally isotropic turbulence

$$F(V)=2k\rho Ve^{-k\rho(V-\bar{u})^2}Q(2k\rho V\bar{u}), \quad \ldots\ldots(2)$$

where
$$Q(x)=e^{-x}J_0(ix) \quad \ldots\ldots(3)$$

and
$$\frac{1}{2k\rho}=\frac{\overline{u'^2}+\overline{v'^2}}{2}=\overline{u'^2}=E' \quad \ldots\ldots(4)$$

u' being the variation of u, i. e. that of the component of V in the direction of the mean wind, and v' being the variation of V in the direction perpendicular to u. Though formulae are given by some authors[8),9)] to obtain E' and $\bar{u}$ (mean value of u) from measured velocity V, we can, for the sake of brevity, put after Best[1)]

$$\overline{V'^2}=\overline{(\bar{V}-V)^2}=\overline{u'^2}=E' \quad \ldots\ldots(5)$$

and
$$\bar{V}=\bar{u}, \quad \ldots\ldots(6)$$

which seem to give good approximations for the present purpose. Making use of values of $\bar{V}$ and $\overline{V'^2}$ obtained from the experiment and (5) and (6) we can evaluate $F(V)$ from (2). (Values of the function $Q(x)$ are obtained from Hesselberg and Björkdal's paper). Smooth curves drawn in fig. 16 are theoretical $F(V)$ thus obtained. Each agreement with histograms is good, and the fundamental assumptions of the theory, i. e. two dimensional isotropy and normal distribution of u and v will be accepted as valid in this case.

(Ⅳ) Frequency distribution of the variation of wind direction.

The frequency distribution of wind direction is obtained by enumerating number of occurrences of direction in each direction intervals of $3\times\frac{360°}{64}$ around the mean, and is shown in fig. 16 in histogram as in the case of wind velocity. It is clear from the figure that the dispersion becomes small in the stable atmosphere——so small that the adoption of direction interval of $3\times\frac{360°}{64}$ is not appropriate——but the height dependence is not distinct.

The theoretical distribution of wind direction was treated by Ertel[5], who obtained for the frequency distribution function in the case of the 2-dimensionally isotropic turbulence

$$F(\varphi)=\frac{e^{-k\rho\bar{u}^2}}{2\pi}+\frac{1}{2\sqrt{\pi}}e^{-k\rho\bar{u}^2\sin^2\varphi}\sqrt{k\rho}\,\bar{u}\cos\varphi\{1+\Phi(\sqrt{k\rho}\,\bar{u}\cos\varphi)\},\ldots\ldots(7)$$

where

$$\Phi(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

and φ is the deviation angle of the wind vector from the mean wind $\bar{u}$ and the meaning of k and ρ is the same as in (4). Each calculated F(φ) is shown in fig. 16 with smooth curve. The agreement with the experiment is good in some cases, but in other cases it is not in the sense of the χ-square test. Generally speaking, the agreement is good in unstable condition, and not good in stable one.

The poor agreement seems to the author to be explained by the fact that dispersions of velocity deviation, $\overline{u'^2}$ and $\overline{v'^2}$, are not the same, because the field on which the experiment was made had the inclination about 4° to NNW as already described and at the sunset the katabatic wind set in which flew down the slope in masses of about several hundred meters in diameter and consequently made $\overline{u'^2}$ (turbulent component parallel to the mean wind) larger than $\overline{v'^2}$ (turbulent component perpendicular to the mean wind). The theoretical distribution of wind direction with different dispersions was treated by Wagner[55], and the obtained frequency distribution is given by

$$F(\varphi)=\frac{\kappa}{2\pi}\frac{e^{-\frac{\bar{u}^2}{2\overline{u'^2}}}}{1-(1-\kappa^2)\cos^2\varphi}\{1+\sqrt{\pi}\,\xi e^{\xi^2}[1+\Phi(\xi)]\}\ \ldots\ldots(9)$$

where $\kappa=\frac{\sqrt{\overline{v'^2}}}{\sqrt{\overline{u'^2}}}$, $\xi=\frac{\bar{u}}{\sqrt{2\overline{u'^2}}}\cdot\frac{\kappa\cos\varphi}{\sqrt{1-(1-\kappa^2)\cos^2\varphi}}$ and $\Phi(x)$ is the same as in (8). In fig. 17 are shown caluculated distributions from (9) for $\kappa=1.0$, 0.5 and 0.2 with actually obtained histogram at the 5 m. level of the experiment No. 5. From the figure it is clear that $\kappa=0.3$ will explain the actual result well.

As the theoretical distribution of velocity with equall dispersions at the 5 m. level of the experiment No. 5. has been recognized to agree fairly well with the experiment as is seen in fig. 16, the next step is to examine whether the

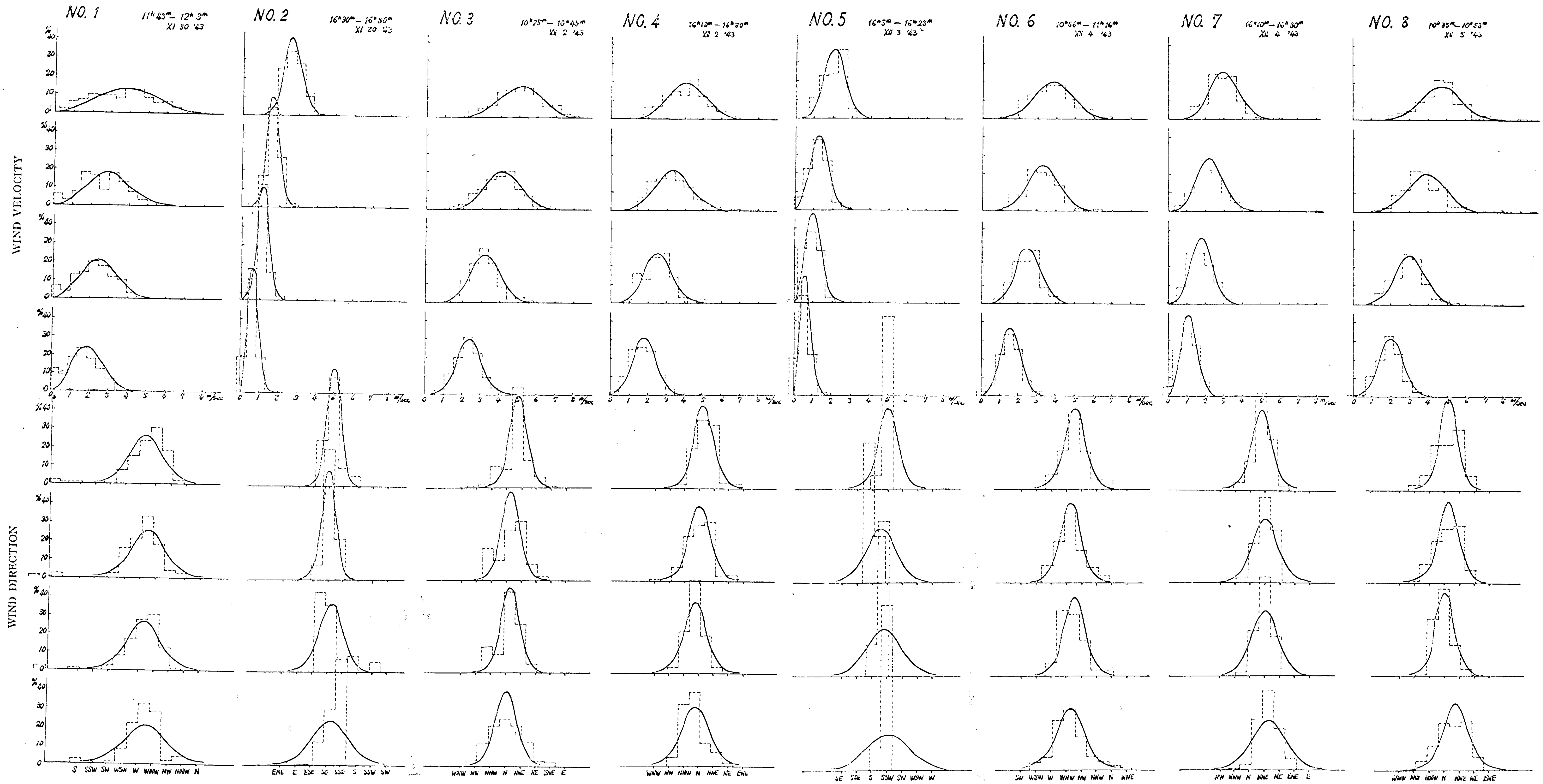

Fig. 16. Frequency distributions of wind velocity and direction with theoretical curves.

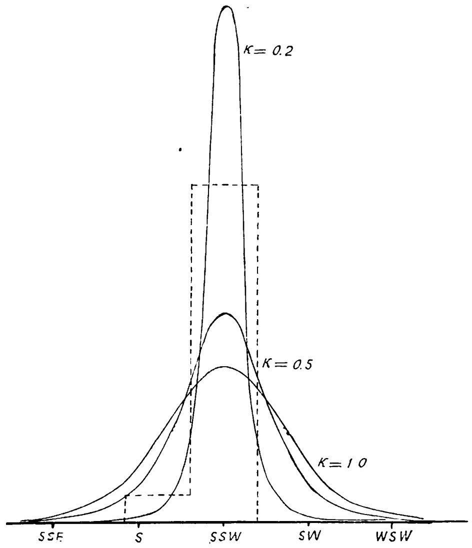

Fig. 17. Frequency distribution of wind direction for z=5 m., No. 5.

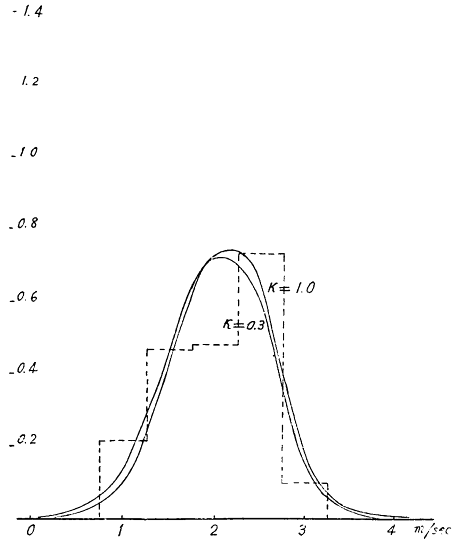

Fig. 18. Frequency distribution of wind velocity for z=5 m., No. 5.

theoretical distribution of velocity with different dispersions for $\kappa=0.3$ shows only a slight deviation from that of equal dispersions. The theoretical distribution of wind velocity with different dispersions was also deduced by Wagner as follows:

$$F(V)=\frac{V e^{-\frac{\bar{u}-V}{2u'^2}}}{\kappa\, u'^2}\left[S_0(\mu)S_0(\nu)+2\sum_{m=1}^{\infty}S_m(\mu)S_{2m}(\nu)\right], \quad \ldots\ldots\ldots\ldots (10)$$

where $\mu=\frac{V^2(1-\kappa^2)}{2u'^2\,\kappa^2}$, $\nu=\frac{\bar{u}\,V}{u'^2}$ and $S_m(x)=\frac{J_m(ix)}{i^m}\,e^{-x}$, $J_m(y)$ being the Bessel Function of the m-th order. Wagner gave values of S_1, S_2, S_3, S_4, S_5, S_6, S_8 and S_{10} for $x=0\sim40$ in a table. But the poor convergency in this case of the series has made the author give up (10) and calculate numerically by the original formula

$$F(V)=\frac{V}{2\pi\kappa u'^2}\int_0^{2\pi} e^{-\frac{V^2}{2u'^2}\left\{\left(\cos\varphi-\frac{u}{V}\right)^2+\frac{\sin^2\varphi}{\kappa^2}\right\}}d\varphi. \quad \ldots\ldots\ldots\ldots (11)$$

In fig. 18 are shown two theoretical curves, one for $\kappa=1$ calculated from (2) and the other for $\kappa=0.3$ obtained from (11), with experimentally obtained histogram. Both curves show only a slight deviation from each other, and our presumption that the disagreement in the frequency distribution in direction is due to the unequalness of the dispersions seems to be confirmed.

Recently Koo[21] has deduced theoretical distributions of wind velocity and direction taking into account the correlation between turbulent components of wind. But to assume a correlation between u' and v' in our case is to accept the predominance of a definite sense of the rotation of vortices with vertical axes in the surface layer which seems improbable, so it may be unnecessary to consider the case with non-zero correlation.

(V) Coefficient of correlation

From the measured velocity two sorts of coefficient of correlation are calculated ——one is the coefficient of correlation (R_y or Ru_{z1}, u_{z2}) between velocities at two points separated by y along the vertical and the other is the coefficient of correlation (R_t) between the velocity at a point and the velocity at the same point but at time t later. Obtained values of R_y are shown after the manner of Schmidt[34] as follows:

Values of Ru_{z1},u_{z2}

(1) unstable, $\overline{V}_5=3.47$ m./sec.

5			
	0.868		
2		0.821	
	0.816		0.781
1		0.756	
	0.876		
0.5			

(2) stable, $\overline{V}_5=2.69$ m./sec.

5			
	0.627		
2		0.363	
	0.235		0.561
1		0.458	
	0.400		
0.5			

(3) unstable, $\overline{V}_5=4.97$ m./sec.

5			
	0.772		
2		0.693	
	0.813		0.685
1		0.659	
	0.725		
0.5			

(4) stable, $\overline{V}_5=4.01$ m./sec.

5			
	0.769		
2		0.704	
	0.846		0.638
1		0.697	
	0.738		
0.5			

(5) stable, $\overline{V}_5=2.01$m./sec.

5			
	0.713		
2		0.188	
	0.430		0.653
1		0.538	
	0.433		
0.5			

(6) unstable, $\overline{V}_5=3.74$ m./sec.

5			
	0.803		
2		0.732	
	0.789		0.644
1		0.710	
	0.670		
0.5			

(7) stable, $\overline{V}_5=2.89$ m./sec.

5			
	0.662		
2		0.649	
	0.731		0.561
1		0.576	
	0.631		
0.5			

(8) unstable, $\overline{V}_5=4.65$ m./sec.

5			
	0.726		
2		0.598	
	0.598		0.658
1		0.798	
	0.678		
0.5			

For the sake of comparison values at the corresponding heights are extracted from Schmidt's paper as follows:

Group I. $\overline{V}_4=2.4$ m./sec.

5		
	0.78	
2		0.69
	0.83	
1		

Group II. $\overline{V}_4=3.8$ m./sec.

5		
	0.42	
2		0.22
	0.55	
1		

It is interesting to note that, though instruments and time scales applied are quite different in Schmidt's and our cases, magnitude of values are approximately the same except in Schmidt's Group II where they are somewhat small. Schmidt describes that values of the correlation coefficient decrease as the mean velocity becomes large, but it is not clear in our case where the effect of stability of the air layer seems to cover more the decrease.

It is clear in our case that values of the correlation coefficient become small as the air layer becomes more stable, which explains the fact that the air layer at different heights generally tends to flow indepenently as the stability becomes large. Shiotani also remarks that the instability of the air layer makes the value of R_y larger.

As to R_t, obtained values are shown in fig. 19 in correlogram. In general R_t seems to become zero for $t=40$ or 50 sec., but in stable atmosphere, e. g. in No. 2 or No. 5, R_t does not reach zero even for $t=170$ sec. As $\int_0^\tau R_t dt$ (τ means the value of t when $R_t=0$) is considered to express a time scale of the predominant eddies, our result shows clearly that the effect of stability tends to make the predominant eddies larger, or eddies of smaller size drop out in the stable atmosphere.

In Shiotani's experiment R_t becomes already zero for $t=10$ or 20 sec. The reducation of this R_t value in Shiotani's case will be ascribed to the small time scale of the experiment which he adopted, e. g. 0.4 or 0.5 sec., whereas in our case it is 10 sec. as already described.

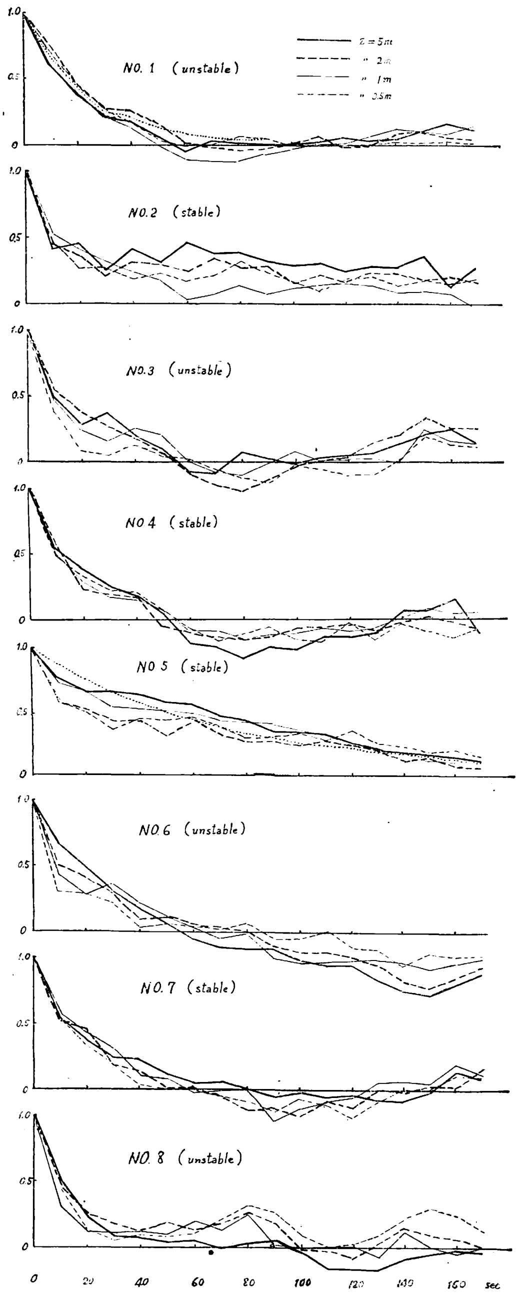

Fig. 19. Correlation coefficients, R_t.

(VI) Coefficient of horizontal mixing, K_h

Coefficient of horizontal mixing may be obtained from various method. Here we have calculated K_h either by Lettau's formula ((26) p. 180)

$$K_h = l\overline{v'}, \quad \ldots\ldots\ldots\ldots\ldots (12)$$

or by Taylor's formula

$$K_h = \overline{v'^2}\int_0^\tau R_t dt^*, \quad \ldots\ldots (13)$$

and plotted them with z in fig. 20. It is interesting to note that the coefficient increases with z but the increase is smaller than linear, whereas in Part I we have seen that eddy viscosity or coefficient of vertical mixing increases almost

* This formula was derived for the Lagrangian coefficient of correlation $R\xi$, and not for the Eulerian coefficient R_t, but can be applied to obtain the approximate magnitude of K_h.

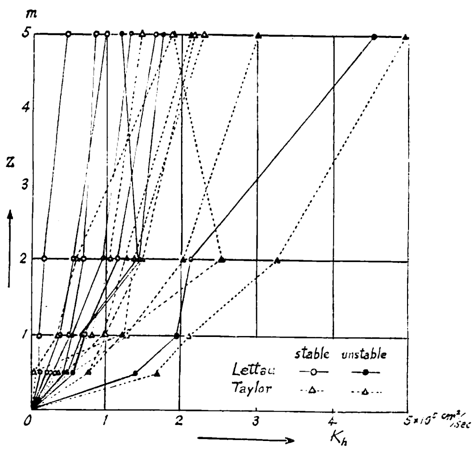

Fig. 20. Coefficient of horizontal mixing.

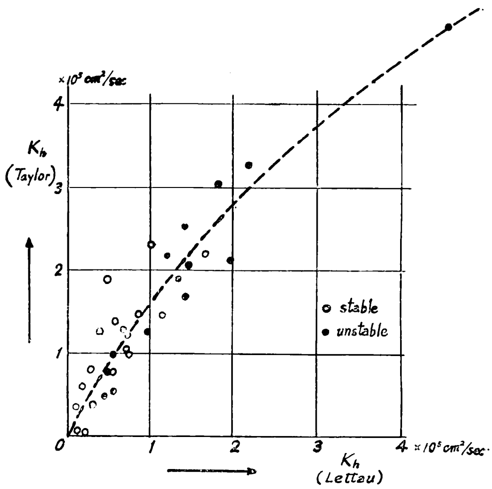

Fig. 21. Coefficient of horizontal mixing calculated by Taylor's formula and Lettau's formula.

linearly with z.

In fig. 21 K_h calculated by (12) is plotted against K_h calculated by (13). From the figure it is seen that K_h calculated by Lettau's formula is smaller by about 30% than K_h calculated by Taylor's formula, and especially in the stable atmosphere.

§ 4. Summary

In view of the fact that published results on the irregular or turbulent component of wind are only a few, the author describes his own experiment in which following results are found:

(1) The turbulent energy $\overline{V'^2}$ increases with height, the rate of increase being somewhat larger than lnz. The stratification of the air layer may have some effect on the rate of increase but it is difficult to find out any law from the experiment except that each value of energy itself becomes smaller with stability.

(2) The relative intensity of turbulence $\sqrt{\overline{V'^2}}\,/\,\overline{V}$ decreases with height. The decrease is very steep in the layer near the ground but soon becomes slight or almost constant as the height increases.

(3) The frequency distribution of the variation of wind velocity can be explained fairly well by the Hesselberg and Björkdal's theory in which 2-dimensional isotropy and normal distribution of components of the variation of wind velocity are assumed.

(4) The frequency distribution of the variation of wind direction can be explained by Ertel's theory in unstable condition but not in stable one. This may be due to the katabatic wind which flew down the field, and Wagner's

theory with different dispersions proves to be in agreement with the results both for wind direction and velocity.

(5) Two sorts of coefficient of correlation between velocities are calculated. One, the coefficient between velocities at two points separated by y along the vertical, is shown to decrease as the stability becomes large, and the other, the coefficient between the velocity at a point and the velocity at the same point but at time t later, seems to decrease with t more gradually to zero as the air layer becomes more stable; and probably this will explain the fact that the effect of stability tends to make the predominant eddies larger or eddies of smaller size drop out in the stable atmosphere.

(6) Coefficient of horizontal mixing are calculated both by Lettau's formula and by Taylor's formula and it is shown that the former gives about 30% smaller values than the latter. The coefficient calculated by either of the two formula seems to increase with z smaller than linear in cotrast to the almost linear increase with z of eddy viscosity or coefficient of vertical mixing.

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地面附近の乱流について

武　田　京　一

森林に対する風害，防風林或は森林火災の研究は近年著しく進歩しつゝあるが，その研究及び対策には平均風速のみならず風のもつ性質，例えば平均風速の垂直分布，風の乱れ等も考慮される様になつた。

然しその為には吾々は先ず是等の風の諸性質を明らかにしておく必要がある。以下かつて著者の行つた実験及び他の著者の実験を基として地面附近の風の性質を調べることにする。

地面附近の風の性質の研究には次の三通りの方法が考えられる。

1）理論的方法

尤もらしい基礎仮定から出発して平均風速，乱れの垂直分布その他を導出するもの，現在迄に若干の試みがなされているが，目下乱流理論そのものが急激な発達の段階にあるので気象への応用は未だしの感があり，一方従来の理論は新しい乱流理論の立場から見ると極めて不十分なものと見られる。

2）実験的方法（一）

平均風速の垂直分布を求めそれからよく知られた関係式を用いて混合距離，渦動粘性係数等を導出しようとするもの，この方法では平均風速の垂直分布を出来るだけ正確に知る必要がある。然し平均風速の測定は比較的容易であるから従来多くの人によつて試みられて来た。

3）実験的方法（二）

風の変動の垂直分布の測定から風の他の性質を調べんとする方法。この方法は極めて興味のある方法ではあるが風の変動の測定は容易でないので余り行われていない。

本報告の目的

現在の所妥当と思われる理論を展開することが困難であつたので主として実験的方法に依つた。本報告の第一部に於いては平均風速垂直分布の問題を取扱い従来の多くの著者（現著者をも含む）の実験的並びに理論的研究を総括し，最も信頼するに足ると思われる平均風速の垂直分布の式を提出する。そして之を用いて混合距離，渦動粘性係数，蒸発公式等を取扱う。第二部に於いては風の変動に関する問題を取扱う。但し風の変動に関する測定結果は従来余り行われていないので著者自身の観測結果について述べ，特に安定度との諸関係を明らかならしめんとした。

結　　果

第一部

1）断熱大気中に於いては殆んど総ての場合に

$$u \sim lnz$$

の如き風の分布式を採用すれば十分であること，又実験（粗度の附近に於ける）がこの分布から偏倚を生ずる場合に於いてのみ

$$u \sim ln(z \pm d)$$

を使用すればよいことが示された。

2）粗度の表面に於いて u が実際の風速に等しくなる如く調節される積分常数 z_0 は安定度と共に変ることが示された。即ち簡単な対数法則

$$u = a ln \frac{z}{z_0}$$

が採用されるならば z_0 は安定度と共に増大する。然し対数法則からの偏倚を表わす法則が採用されるならば z_0 は安定度と共に減少する。そして多分この方が実際であろうと思われる。

3）Deacon の新しい分布式中のパラメーター a 及びそれから得られる渦動粘性係数 K の式は誤りであることが示され，訂正された式が提出された。

4）訂正された K の式を使用すると Pasquill の実験に於いて $K = K_v$（水蒸気に対する渦動拡散係数）を実験的に示すことが出来る。（Pasquill は Deacon の提出した K の式を使つた為それが出来なかつた）。

5）現在の所 Deacon の速度分布式

$$u = \frac{a}{1-\beta}(z^{1-\beta} - z_0^{1-\beta})$$

が最も簡単で且つ実験に最もよく適合すると考えられる。（但し z_0 は Deacon とは異なり安定度によつて変わる。即ち安定度と共に減少すると考える）。そして以後の考察の基礎式に採用される。

6）Deacon の分布式を採用すると混合距離及び渦動粘性係数は次の如くなる。

$$l = 0.4\,h(\beta)^{\beta} z$$

及び

$$K = 0.4 v_{*} h(\beta) z^{\beta}.$$

ここで安定度のみによつて変ると考えられるパラメーター h は Pasquill の実験から求められる。

7）v_{*}/U の安定度に対する変化を Pasquill の実験から求めた所 $R_i = 0$ で極小となつた。この結果は少し奇妙なものゝ如く思われるので将来更に他の資料に就いて検討する必要があろう。

8）上に定義されたパラメーター $h(\beta)$ は Thornthwaite の蒸発公式中にも生ずることが示された。そして大きな面積から正確な蒸発量を求める Thornthwaite の方法は非断熱大気の場合には $h(\beta)$ が他の方法例えば τ_0 或は気温と輻射量等から求められない以上適用し得ないことが注意された。

第二部

風の変動成分に関する発表された結果は少いので著者自身の実験に就いて述べる。得られた結果は次の如くである。

1）乱流エネルギー $\overline{V'^2}$ は高度と共に増大する。増大の割合は lnz よりは幾分大である。気温の垂直分布の影響は，エネルギーの値自身が安定大気中で小さくなる以外には，実験からは明瞭でなかつた。

2）乱流の相対強度 $\sqrt{\overline{V'^2}}/\overline{V}$ は高度と共に減少する。その減少は地面附近では極めて急激であるが，高さと共に直きに緩慢或は殆んど一定となる。

3）風速変動の頻度分布は変動成分の二次元等方性及び正規分布を仮定した Hesselberg 及び Björkdal の理論によつて可成りよく説明される。

4）風向変動の頻度分布は不安定条件の場合には Ertel の理論でよく説明出来るが，安定条件の場合には説明出来ない。之はその際に存在したカタバ風の為であると考えられる。そして方向によつて異なれる分散をもつた Wagner の理論が風速並びに風向の結果をよく説明することが示された。

5）風速変動成分の間の二種類の相関係数 R_y 及び R_t が求められた。垂直方向に y だけ隔つた二点に於ける速度成分の間の相関係数 R_y は安定度が増すと共に減少する。之は安定大気中では上下の空気層が夫々独立に流れようとする傾向を示すものであろう。又一点に於ける速度と同じ点に於ける時間 t 後の速度との間の相関係数 R_t は t と共に減少するが，安定大気の場合にはその減少がより緩慢である。そして之は多分安定大気中では卓越する渦が大きくなる，或は小さな渦が消失する事を説明するものであろう。

6）水平拡散の係数が Lettau 及び Taylor の方法によつて求められた。そして前者の方が後者より約 30% 小さな値を与えることが示された。何れの方法によるも求められた係数は高度に対して直線的程には増大しないことが認められる（之に対して渦動粘性係数或は垂直拡散の係数は高度と共に殆んど直線的に増大する）。